

Separace proměnných - příklady 1

① $y' = \frac{y}{x}$, $\frac{dy}{dx} = \frac{y}{x}$, $\frac{dy}{y} = \frac{dx}{x}$ $y \neq 0$
 $x \neq 0$ $y=0$ řešení
 $(0' = \frac{0}{x})$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$e^{(\ln|y|)} = e^{(\ln|x| + \ln C_1)}, \quad C_1 > 0$$

$$|y| = C_1 \cdot |x|$$

$$y = C_2 x, \quad C_2 \neq 0 \quad (C_2 = \pm C_1)$$

$$\left[y = Cx, \quad x \neq 0, \quad C \in \mathbb{R} \right]$$

② $y' = \frac{x}{y}$, $\frac{dy}{dx} = \frac{x}{y}$, $y dy = x dx$
 $y \neq 0$

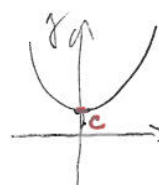
$$\int y dy = \int x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{C}{2} \quad | \cdot 2$$

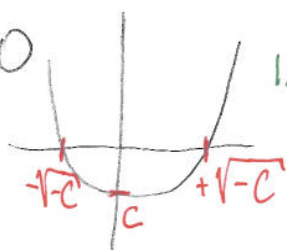
$$y^2 = x^2 + C$$

$$\left[y = \pm \sqrt{x^2 + C}, \quad x^2 + C \geq 0 \right]$$

$y \neq 0$

$C > 0$  $x \in \mathbb{R}$

$C = 0$  $x \in \mathbb{R} \setminus \{0\}$

$C < 0$  $|x| \geq \sqrt{-C}$

③ $y' = -\frac{y}{x}$, $\frac{dy}{dx} = -\frac{y}{x}$, $\frac{dy}{y} = -\frac{dx}{x}$ $y \neq 0$ 2
 $x \neq 0$ [$y \equiv 0$ řešení]

$$\int \frac{dy}{y} = - \int \frac{dx}{x}$$

$$e^{(\ln|y|)} = (-\ln|x| + \ln C_1), \quad C_1 > 0$$

$$|y| = \frac{C_1}{|x|}$$

$$y = \frac{C_2}{x}, \quad C_2 \neq 0 \quad (C_2 = \pm C_1)$$

$$\left[y = \frac{C}{x}, \quad C \in \mathbb{R}, \quad x \neq 0 \right]$$

④ $y' = -\frac{x}{y}$, $\frac{dy}{dx} = -\frac{x}{y}$, $y dy = -x dx$
 $y \neq 0$

$$\int y dy = - \int x dx$$

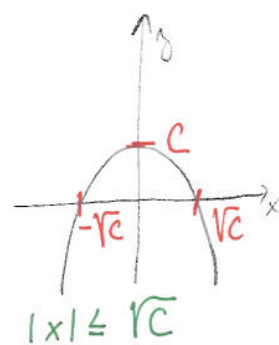
$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{C}{2}, \quad C \in \mathbb{R}$$

$$y^2 = -x^2 + C$$

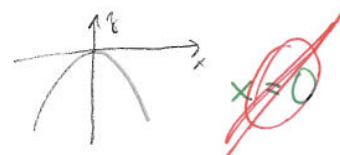
$$\left[y = \pm \sqrt{-x^2 + C}, \quad -x^2 + C \geq 0 \right]$$

$y \neq 0$

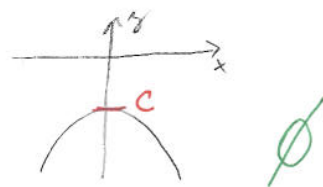
$C > 0$



$C = 0$



$C < 0$



⑤

$$y' = \frac{y-1}{x^2 y^2}$$

 $x \neq 0, y \neq 0$

$$\frac{dy}{dx} = \frac{y-1}{x^2 y^2}$$

$$\frac{y^2}{y-1} dy = \frac{dx}{x^2}$$

3

$$\int \frac{y^2}{y-1} dy = \int \frac{dx}{x^2}$$

$$\left[\begin{array}{l} y-1 \neq 0 \\ y \neq 1 \\ y \neq 7 \text{ Wesen!} \end{array} \right]$$

$$\int \frac{y^2}{y-1} dy = \int \frac{\overbrace{y^2 - y}^0 + \overbrace{y-1+1}^0}{y-1} dy =$$

$$= \int \left(\frac{y(y-1)}{y-1} + \frac{y-1}{y-1} + \frac{1}{y-1} \right) dy =$$

$$= \int \left(y + 1 + \frac{1}{y-1} \right) dy = \int y dy + \int dy + \int \frac{dy}{y-1} =$$

$$= \frac{y^2}{2} + y + \ln|y-1| + \text{konstante}$$

$$\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-1}}{-1} + \text{int. konst.} = -\frac{1}{x} + \text{int. b.}$$

$$\left[\frac{y^2}{2} + y + \ln|y-1| = -\frac{1}{x} + C, \quad C \in \mathbb{R} \right]$$

 $x \neq 0, y \neq 0$
 $y \neq 1$