Písemka	P1–A	ZS-2012

21. listopadu 2012

KMA-MA2AA

VZOR ....

1. Vypočtěte $\lim\limits_{n\to+\infty} \dfrac{3n^2+2n+10}{-n+5}$.

2. Vypočtěte $\lim\limits_{x\to 0} \dfrac{\cos x}{x^2}$.

3. Vypočtěte $\lim\limits_{x\to 0^+} x\ln x$.

4. Vypočtěte první derivaci funkce $f: y = \operatorname{arctg}\left(x^2+\sqrt{x^3\cos x}\right)$.

(1) $$\lim_{x\to+\infty} \frac{3n^2+2n+10}{-n+5} = \lim_{n\to+\infty} \frac{n^{\cancel{2}}\left(3+\frac{2}{n}+\frac{10}{n^2}\right)}{\cancel{n}\left(-1+\frac{5}{n}\right)} = \left[\frac{\infty(3+0+0)}{-1+0} = \frac{3\cdot\infty}{-1}\right] = \underline{\underline{-\infty}}$$

(2) $$\lim_{x\to 0} \frac{\cos x}{x^2} = \left[\frac{1}{0^+}\right] = +\infty$$

(3) $$\lim_{x\to 0^+} x\cdot\ln x = \underbrace{\left[0\cdot(-\infty)\right]}_{ND} = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} = \left[\frac{-\infty}{+\infty}\right] \overset{LP}{=}$$

$$\overset{LP}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^2}{-1} = \lim_{x\to 0^+}(-x) = \underline{\underline{0}}$$

(4) $$\left[\operatorname{arctg}\left(x^2+\sqrt{x^3\cos x}\right)\right]' = \frac{1}{1+\left(x^2+\sqrt{x^3\cos x}\right)^2}\cdot\left(x^2+\sqrt{x^3\cos x}\right)' =$$

$$= \frac{1}{1+\left(x^2+\sqrt{x^3\cos x}\right)^2}\cdot\left[2x+\frac{1}{2\sqrt{x^3\cdot\cos x}}\cdot\underbrace{\left(3x^2\cdot\cos x - x^3\cdot\sin x\right)}_{(x^3\cos x)'}\right]$$

Písemka $\boxed{\text{P1–B}}$ ZS-2012	21. listopadu 2012 $\boxed{\text{KMA-MA2AA}}$	VZOR

1. Vypočtěte $\displaystyle\lim_{n\to+\infty}\frac{2n^2-2n-5}{-3n+7}$.

2. Vypočtěte $\displaystyle\lim_{x\to+\infty}\frac{\sin x}{x}$.

3. Vypočtěte $\displaystyle\lim_{x\to0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$.

4. Vypočtěte první derivaci funkce $f: y=\ln\left(x^2+5\sqrt{x^3\sin x}\right)$.

① $$\lim_{n\to+\infty}\frac{2n^2-2n-5}{-3n+7}=\lim_{n\to+\infty}\frac{n^{\cancel{2}}\left(2-\frac{2}{n}-\frac{5}{n^2}\right)}{\cancel{n}\left(-3+\frac{7}{n}\right)}=\left[\frac{\infty(2-0-0)}{-3+0}=\frac{\infty\cdot 2}{-3}=\frac{\infty}{-3}\right]=-\infty$$

② $$\lim_{x\to+\infty}\frac{\sin x}{x}=\left[\frac{\text{omezená}}{\infty}\right]=0$$

③ $$\lim_{x\to0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)=\cancel{\lim_{x\to0^+}}\left[\frac{1}{0^+}-\frac{1}{0^+}=\underset{ND}{\infty-\infty}\right]=$$

$$=\lim_{x\to0^+}\frac{\sin x-x}{x\cdot\sin x}=\left[\frac{0}{0}\right]\overset{LP}{=}\lim_{x\to0^+}\frac{\cos x-1}{1\cdot\sin x+x\cdot\cos x}=\left[\frac{1-1}{0+0}=\frac{0}{0}\right]\overset{LP}{=}$$

$$\overset{LP}{=}\lim_{x\to0^+}\frac{-\sin x-0}{\cos x+1\cdot\cos x-x\sin x}=\left[\frac{0}{1+1-0}=\frac{0}{2}\right]=0$$

④ $$\left[\ln\left(x^2+5\sqrt{x^3\sin x}\right)\right]'=\frac{1}{x^2+5\sqrt{x^3\sin x}}\cdot\left(x^2+5\sqrt{x^3\sin x}\right)'=$$

$$=\frac{1}{x^2+5\sqrt{x^3\sin x}}\cdot\left[2x+\frac{5}{2\sqrt{x^3\sin x}}\cdot\underbrace{\left(3x^2\cdot\sin x+x^3\cdot\cos x\right)}_{(x^3\sin x)'}\right]$$