Písemka P1–C ZS-2012	30. listopadu 2012 KMA-MA2AA	VZOR

1. Vypočtěte:

$(x^n)' = n \cdot x^{n-1}$

$(\sin 2x)' = (\cos 2x) \cdot 2$

$(\ln x)' = \frac{1}{x}$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(3^x)' = 3^x \cdot \ln 3$

$(\cos x)' = -\sin x$

$(e^x)' = e^x$

$(\log_5 x)' = \frac{1}{x \cdot \ln 5}$

$(\operatorname{cotg} x)' = \frac{-1}{\sin^2 x}$

$(\operatorname{tg} 3x)' = \frac{3}{\cos^2 3x}$

2. Vypočtěte $\lim_{n\to+\infty} \frac{2n^3-2n-5}{-6n+7}$. $= \left[\frac{\infty}{-\infty}\right] = \lim_{n\to+\infty} \frac{n^{\cancel{3}2}\left(2-\frac{2}{n^2}-\frac{5}{n^3}\right)}{\cancel{n}\left(-6+\frac{7}{n}\right)} = \left[\frac{\infty(2-0-0)}{-6+0} = \frac{\infty}{-6}\right] = -\infty$

3. Vypočtěte $\lim_{x\to 1} x\cos\left(\pi\, e^{12x^5-17x+5}\right) = \left[1\cdot\cos(\pi\cdot e^0) = \cos\pi\right] = -1$

4. Vypočtěte druhou derivaci funkce $f: y = \ln\left(x^2+5\sin x\right)$. $y' = \frac{1}{x^2+5\sin x}\cdot(2x+5\cos x)$

$$y'' = \left(\frac{2x+5\cos x}{x^2+5\sin x}\right)' = \frac{(2-5\sin x)(x^2+5\sin x)-(2x+5\cos x)(2x+5\cos x)}{(x^2+5\sin x)^2}$$

5. Vypočtěte $\lim_{x\to 1}\left(\frac{\cos(\pi x)+1}{(x-1)^2}\right) = \left[\frac{-1+1}{0^2} = \frac{0}{0}\ \mathrm{ND}\right] \overset{LP}{=} \lim_{x\to 1}\frac{-\pi\sin\pi x}{2(x-1)\cdot 1} = \left[\frac{-\pi\cdot 0}{2\cdot 0} = \frac{0}{0}\right]$

$$\overset{LP}{=} \lim_{x\to 1}\frac{-\pi^2\cos\pi x}{2} = \left[\frac{-\pi^2\cdot(-1)}{2}\right] = \frac{\pi^2}{2}$$