(3)

Písemka $\boxed{\text{P2-C}}$ ZS-2012	14. prosince 2012 $\boxed{\text{KMA-MA2AA}}$	VZOR ....

1. Vyšetřete průběh funkce $f : y = \dfrac{x^2}{x-1}$. *(15 bodů)*

2. Vypočtěte $\displaystyle\int_0^1 x\,\mathrm{e}^x\,\mathrm{d}x$. *(5 bodů)*

3. Vypočtěte $\displaystyle\int \left(\frac{3}{\sin^2 x} - 5^{\sin x}\cdot\cos x\right)\mathrm{d}x$. *(8 bodů)*

(3) $$\int\left(\frac{3}{\sin^2 x} - 5^{\sin x}\cdot\cos x\right)dx =$$

$$= 3\cdot\int\frac{dx}{\sin^2 x} - \int 5^{\sin x}\cos x\,dx =$$

$$= 3\cdot(-\cot x) - \frac{5^{\sin x}}{\ln 5} + C$$

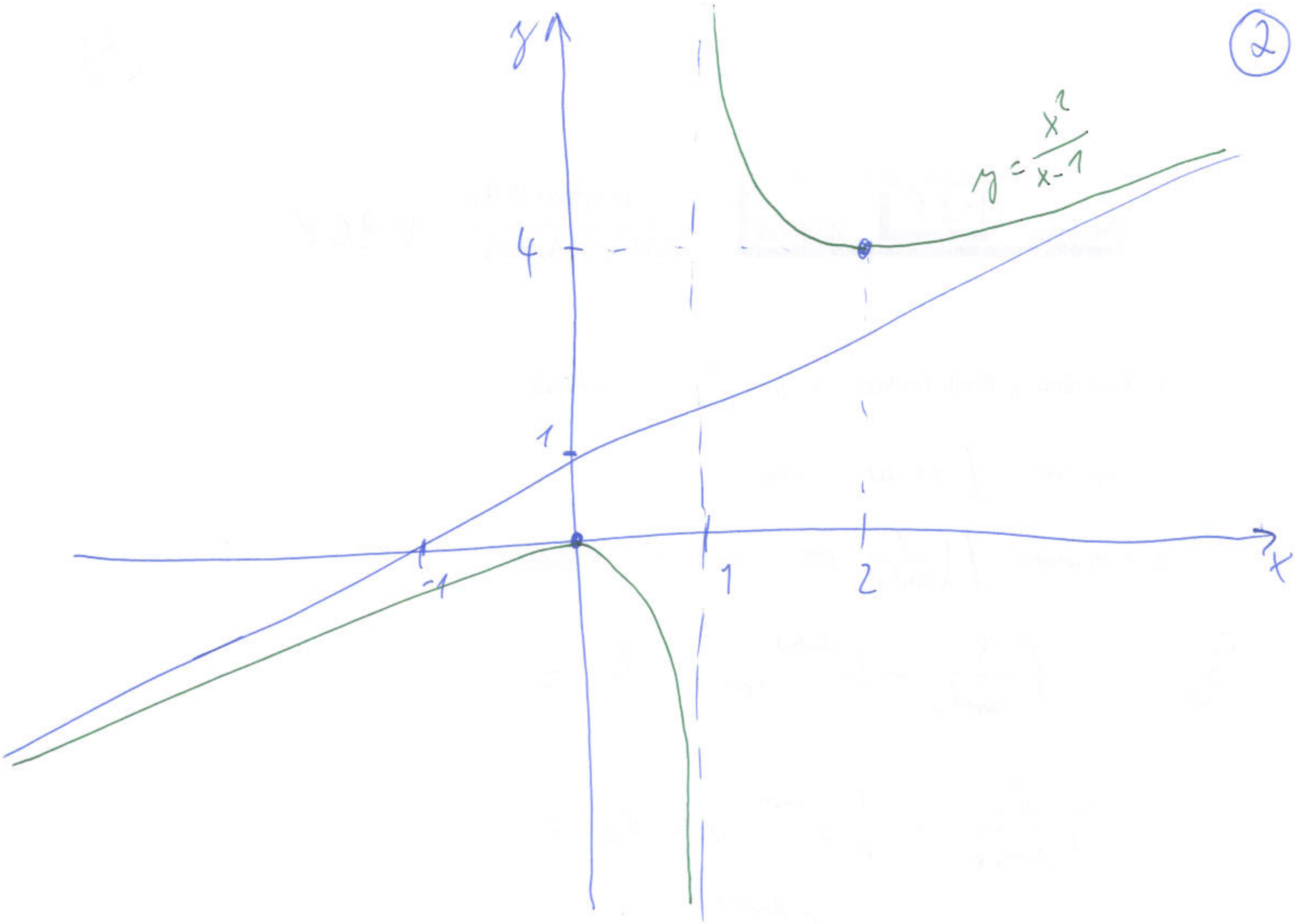
$$\int 5^{\sin x}\cos x\,dx = \left[\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}\right] = \int 5^u\,du =$$

$$= \frac{5^u}{\ln 5} + C = \frac{5^{\sin x}}{\ln 5} + C$$

(2) $$\int_0^1 x\cdot e^x\,dx = \left[\begin{array}{ll} u = x & v' = e^x \\ u' = 1 & v = e^x \end{array}\right] = \left[x\cdot e^x\right]_{x=0}^{1} - \int_0^1 e^x\,dx =$$

$$= \left[x\cdot e^x\right]_{x=0}^{1} - \left[e^x\right]_{x=0}^{1} = (1\cdot e^1 - 0) - (e^1 - e^0) = e - e + 1 = \underline{\underline{1}}$$

②



Písemka **P2-C** ZS-2012

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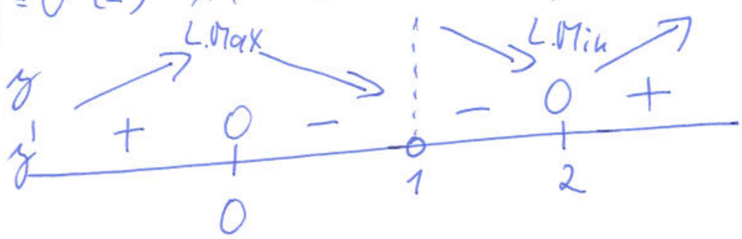
1 $f: y = \dfrac{x^2}{x-1}$ $\quad D(f) = \mathbb{R} \setminus \{1\}$ $\quad$ ani sudá, ani lichá, ani periodická

$$y' = \frac{2x(x-1) - x^2\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2} \qquad D(f') = \mathbb{R}\setminus\{1\}$$

$y' = 0 \iff x(x-2) = 0 \qquad x_1 = 0,\ x_2 = 2$

L.Max $f(0) = \dfrac{0^2}{0-1} = 0$

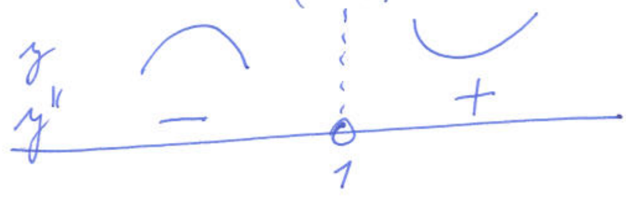
L.Min $f(2) = \dfrac{2^2}{2-1} = 4$



$D(f'') = \mathbb{R}\setminus\{1\}$

$$y'' = \frac{(2x-2)(x-1)^{\cancel{2}} - (x^2-2x)\cdot 2\cdot\cancel{(x-1)}}{(x-1)^{\cancel{4}3}} = \frac{2x^2-2x-2x+2-2x^2+4x}{(x-1)^3} = \frac{2}{(x-1)^3}$$

$y'' = 0$ nemá řešení



nemá infl.

asymptoty

svislé: $\displaystyle\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^2}{x-1} = \left[\frac{1^2}{0^-}\right] = -\infty \implies$ as. $x = 1$

$\displaystyle\lim_{x\to 1^+} \frac{x^2}{x-1} = \left[\frac{1^2}{0^+}\right] = +\infty$

šikmé: $\displaystyle k = \lim_{x\to -\infty} \frac{f(x)}{x} = \lim_{x\to -\infty} \frac{x}{x-1} = 1$

$$q = \lim_{x\to -\infty} (f(x) - 1\cdot x) = \lim_{x\to -\infty} \left(\frac{x^2}{x-1} - x\right) = \lim_{x\to -\infty} \frac{x^2-x^2+x}{x-1} = \lim_{x\to -\infty} \frac{x}{x-1} = 1$$

as. $y = 1\cdot x + 1 = x+1$ $\quad$ Pro $x \to +\infty$ vyjde stejná as.