Písemka **P2-D** ZS-2012	18. prosince 2012 KMA-MA2AA	VZOR ....

1. Vyšetřete průběh funkce $f : y = \dfrac{x^3}{x^2-3}$. *(15 bodů)*

2. Vypočtěte $\displaystyle\int_0^{\pi/2} x^2 \sin x \,\mathrm{d}x$. *(6 bodů)*

3. Vypočtěte $\displaystyle\int \operatorname{tg} x \,\mathrm{d}x$. *(5 bodů)*

(1) $f: y = \dfrac{x^3}{x^2-3}$, $D(f) = \mathbb{R} \setminus \{\pm\sqrt{3}\}$ | $\lim\limits_{x\to-\infty} f(x) = -\infty$, $\lim\limits_{x\to+\infty} f(x) = +\infty$

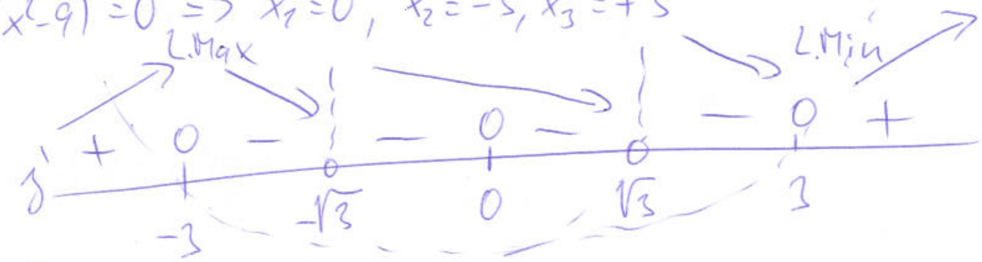
$f(-x) = -\dfrac{x^3}{x^2-3} \Rightarrow$ lichá fce

$$y' = \frac{3x^2(x^2-3) - x^3(2x)}{(x^2-3)^2} = \frac{3x^4 - 9x^2 - 2x^4}{(x^2-3)^2} = \frac{x^4-9x^2}{(x^2-3)^2} = \frac{x^2(x^2-9)}{(x^2-3)^2} \quad D(f') = D(f)$$

$y' = 0 \Leftrightarrow x^2(x^2-9) = 0 \Rightarrow x_1 = 0,\ x_2 = -3,\ x_3 = +3$

L.Min $f(3) = \dfrac{27}{9-3} = \dfrac{9}{2}$

L.Max $f(-3) = -\dfrac{9}{2}$

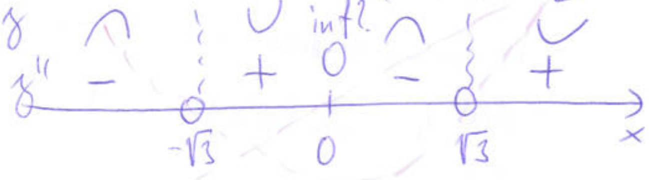


$$y'' = \frac{(4x^3-18x)(x^2-3)^{\cancel{2}\,1} - (x^4-9x^2)\,2\cdot\cancel{(x^2-3)}\cdot 2x}{(x^2-3)^{\cancel{4}\,3}} = \frac{\cancel{4x^5} - 12x^3 - 18x^3 + 54x - \cancel{4x^5} + 36x^3}{(x^2-3)^3} =$$

$D(f'') = D(f)$

$$y'' = \frac{6x^3+54x}{(x^2-3)^3} = \frac{6x(x^2+9)}{(x^2-3)^3}$$

$y'' = 0 \Leftrightarrow 6x(x^2+9) = 0$, $x_1 = 0$, $\emptyset$



konvexní $(-\sqrt{3}, 0)$, $(\sqrt{3}, +\infty)$

konkávní $(-\infty, -\sqrt{3})$, $(0, \sqrt{3})$

inflexní bod $[0, f(0)] = [0, 0]$

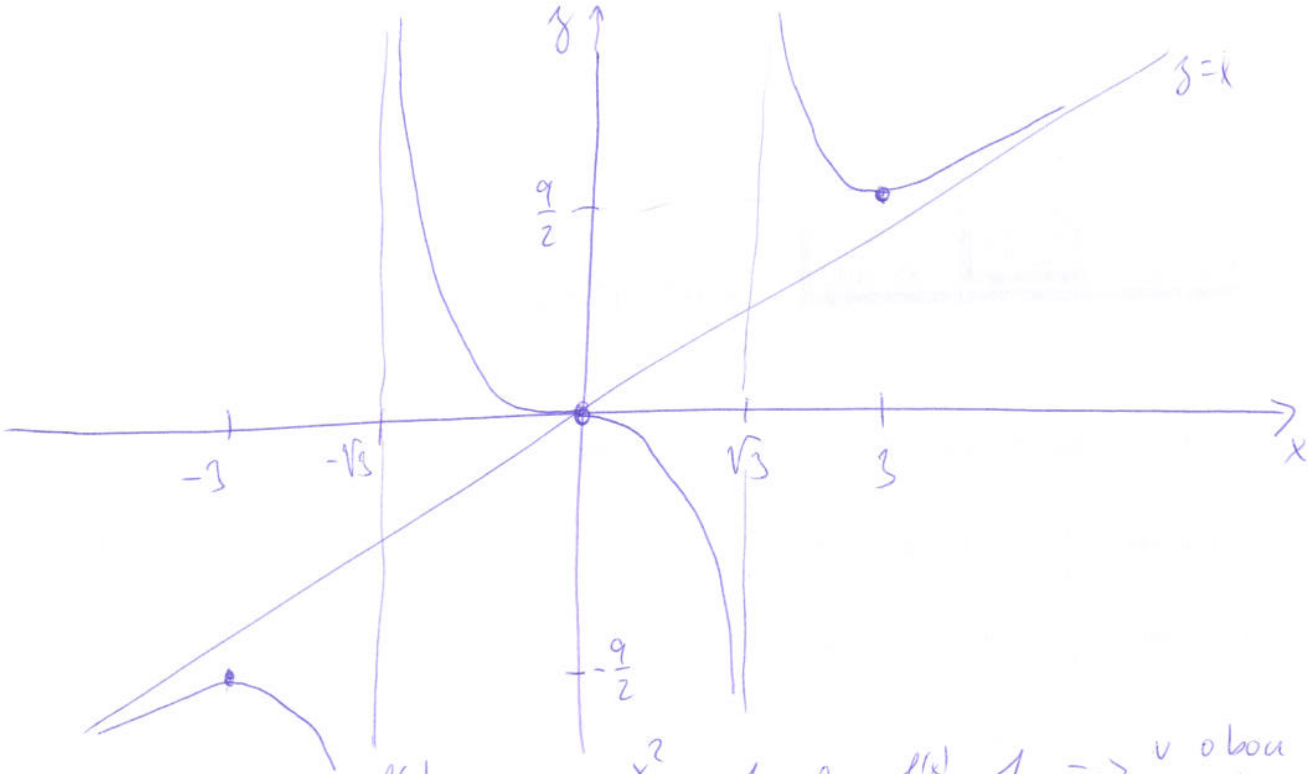
asymptoty a limity:

svislé – $\lim\limits_{x\to-\sqrt{3}^-} f(x) = \left[\dfrac{-3\sqrt{3}}{0^+}\right] = -\infty$, $\lim\limits_{x\to-\sqrt{3}^+} f(x) = \left[\dfrac{-3\sqrt{3}}{0^-}\right] = +\infty$

$\Downarrow$ as $\boxed{x = -\sqrt{3}}$

$\lim\limits_{x\to\sqrt{3}^-} f(x) = \left[\dfrac{3\sqrt{3}}{0^-}\right] = -\infty \Rightarrow$ as $\boxed{x = \sqrt{3}}$, $\lim\limits_{x\to\sqrt{3}^+} f(x) = \left[\dfrac{3\sqrt{3}}{0^+}\right] = +\infty$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x^{\cancel{3}}}{\cancel{x^2}(1-\frac{3}{x^2})} = \left[\dfrac{-\infty}{1-0}\right] = -\infty$, $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^3}{x^2(1-\frac{3}{x^2})} = \left[\dfrac{\infty}{1}\right] = +\infty$



šikmé as.: $\lim\limits_{x\to-\infty}\frac{f(x)}{x} = \lim\limits_{x\to-\infty}\frac{x^2}{x^2-3} = 1$, $\lim\limits_{x\to+\infty}\frac{f(x)}{x} = 1$ => v obou směrech je $k = 1$

$$\lim_{x\to-\infty}(f(x)-kx) = \lim_{x\to-\infty}\left(\frac{x^3}{x^2-3}-1\cdot x\right) = \lim_{x\to-\infty}\frac{x^3-x^3+3x}{x^2-3} = \lim_{x\to-\infty}\frac{3x}{x^2-3} = 0$$

$\lim\limits_{x\to+\infty}(f(x)-kx) = 0$ v obou směrech $q = 0$

Existuje tedy jedna společná as. pro oba směry

$$\boxed{y = x}$$

(2) $$\int_0^{\frac{\pi}{2}} x^2\sin x\,dx = \Big[-x^2\cos x + 2x\sin x + 2\cos x\Big]_{x=0}^{\frac{\pi}{2}} = \left(-\left(\frac{\pi}{2}\right)^2\cdot 0 + 2\cdot\frac{\pi}{2}\cdot 1 + 2\cdot 0\right) - (0\cdot 1 + 0 + 2) = \pi - 2$$

$$\int x^2\sin x\,dx = \begin{bmatrix} u = x^2 & v' = \sin x \\ u' = 2x & v = -\cos x \end{bmatrix} = x^2(-\cos x) - \int 2x(-\cos x)\,dx = -x^2\cos x + 2\int x\cos x\,dx =$$

$$\begin{bmatrix} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{bmatrix} = -x^2\cos x + 2\left[x\sin x - \int\sin x\,dx\right] = -x^2\cos x + 2x\sin x + 2\cos x + C$$

(3) $$\int \mathrm{tg}\,x\,dx = \int\frac{\sin x}{\cos x}\,dx = -\int\frac{-\sin x}{\cos x}\,dx = \left[\begin{matrix}\text{v čitateli} \\ \text{je derivace jmenovatele}\end{matrix}\right] = -\ln|\cos x| + C$$

alternativně (bez vzorečku $\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$)

$$\int\frac{\sin x}{\cos x}\,dx = \begin{bmatrix} u = \cos x \\ du = -\sin x\,dx \end{bmatrix} = \int\frac{-du}{u} = -\int\frac{du}{u} = -\ln|u| + C =$$

$-du = \sin x\,dx$

$$= -\ln|\cos x| + C$$