tabé P1-03 MA2AA

Písemka **P1–C** ZS-2011

8. prosince 2011

KMA-MAF1

VZOR ....

1. Vypočtěte $\lim\limits_{n\to+\infty} \dfrac{3\cdot\sqrt{n+2}\cdot\sin(n^2+2n)}{6n-\sqrt{n}}$.
2. Vypočtěte $\lim\limits_{x\to+\infty} \dfrac{\operatorname{arctg} x}{\operatorname{arccotg} x}$.
3. Vypočtěte $f''(2)$ a $f'''(1)$, je-li $f: y = x\ln x$.
4. Vypočtěte derivaci funkce $f: y = \log_{e^2}\left(\dfrac{1}{e^{25\pi}+x^3\cdot 3^{2x}}\right)$.

(1) $-1 \le \sin(n^2+2n) \le 1$

Pozn.: U limity posl. nelze (alespoň ne přímo) použít l'H. Pravidlo!

$$-1\cdot\frac{3\sqrt{n+2}}{6n-\sqrt{n}} \le \frac{3\sqrt{n+2}\cdot\sin(n^2+2n)}{6n-\sqrt{n}} \le \frac{3\sqrt{n+2}}{6n-\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{3\sqrt{n+2}}{6n-\sqrt{n}} = \left[\frac{\infty}{\infty-\infty}\right] = \lim_{n\to\infty}\frac{3\cdot\cancel{\sqrt{n}}\cdot\sqrt{1+\frac{2}{n}}}{\cancel{\sqrt{n}}(6\sqrt{n}-1)} = \left[\frac{3\sqrt{1+0}}{6\cdot\infty-1}=\frac{3}{\infty}\right] = \underline{0}$$

$$\lim_{n\to\infty} -\frac{3\sqrt{n+3}}{6n-\sqrt{n}} = \ldots = -0 = \underline{0}$$

V o 3 limitách

$$\lim_{n\to\infty}\frac{3\sqrt{n+2}\,\sin(n^2+2n)}{6n-\sqrt{n}} = \underline{\underline{0}}$$

(2) $$\lim_{x\to+\infty}\frac{\operatorname{arc\,tg} x}{\operatorname{arccotg} x} = \left[\frac{\frac{\pi}{2}}{0^+}\right] = \underline{\underline{+\infty}}$$

Pozn.: Nejde o limitu typu $\frac{0}{0}$ nebo $\frac{\infty}{\infty}$, nelze použít LP.

(3) $f'(x) = 1\cdot\ln x + x\cdot\frac{1}{x} = \ln x + 1$, $f''(x) = \frac{1}{x}+0 = \frac{1}{x}$, $f'''(x) = -\frac{1}{x^2}$

$\underline{\underline{f''(2) = \frac{1}{2}}}$, $\underline{\underline{f'''(1) = -1}}$

(4) $$y' = \frac{1}{\frac{1}{e^{25\pi}+x^3\cdot 3^{2x}}\cdot\ln e^2}(-1)\cdot(e^{25\pi}+x^3\cdot 3^{2x})^{-2}\cdot(0+3x^2\cdot 3^{2x}+x^3\cdot 3^{2x}\cdot\ln 3\cdot 2) =$$

$$= \frac{\cancel{(e^{25\pi}+x^3\cdot 3^{2x})}\,1}{\ln e^2 = 2}\cdot\frac{-1}{(e^{25\pi}+x^3\cdot 3^{2x})^{\cancel{2}}}\cdot 1\left(x^2\cdot 3^{2x+1}+2\cdot\ln 3\cdot x^3\cdot 3^{2x}\right) =$$

$$= \underline{\underline{-\frac{1}{2}\,\frac{x^2\cdot 3^{2x+1}+2\ln 3\,x^3\,3^{2x}}{e^{25\pi}+x^3\cdot 3^{2x}}}}$$