

1. V lavici mohou sedět čtyři žáci. Kolikovým způsobem je možno lavici obsadit, máme-li pět žáků a záleží na pořadí míst?

2. Řešte  $|2x + 1| + |1 - x| = 3$ .

3. Vypočtěte

$$\begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$$

4. Vypočítejte inverzní matici  $A^{-1}$  k matici  $A$  a proveďte zkoušku:  $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & -1 & 1 \end{pmatrix}$ .

5. Vyřešte soustavu lineárních rovnic

$$\begin{array}{rcl} x_1 + x_2 + x_3 + x_4 & = & -3 \\ x_1 + x_2 - x_4 & = & -4 \\ x_2 - x_3 + x_4 & = & 5 \end{array}$$

$$\textcircled{1} V_4(5) = \frac{5!}{(5-4)!} = 5! = \underline{\underline{120}} \quad \checkmark$$

$$\textcircled{2} \text{N. B. } x_1 = -\frac{1}{2}, \quad x_2 = 1$$

$$x \quad (-\infty, -\frac{1}{2}) \quad (-\frac{1}{2}, 1) \quad (1, +\infty)$$

$$|2x+1| \quad -2x-1 \quad 2x+1 \quad 2x+1$$

$$|1-x| \quad 1-x \quad 1-x \quad x-1$$

$$\begin{array}{l} |2x+1| + |1-x| = 3 \\ -2x-1+1-x=3 \quad 2x+1+1-x=3 \quad 2x+1+x-1=3 \\ -3x=3 \quad x+2=3 \quad 3x=3 \\ x=-1 \in (-\infty, -\frac{1}{2}) \quad x=1 \quad x=1 \in (1, +\infty) \\ \text{OK} \quad \notin (-\frac{1}{2}, 1) \quad \text{OK} \end{array}$$

$$X = \underline{\underline{\{-1, 1\}}} \quad \checkmark$$

$$\textcircled{3} \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 3 \\ 1 & 1 & 5 & 9 \\ 1 & 2 & 9 & 19 \end{vmatrix} = 1 \cdot (-1)^2 \begin{vmatrix} 0 & 2 & 3 \\ 1 & 5 & 9 \\ 2 & 9 & 19 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 3 \\ 1 & 5 & 9 \\ 0 & -1 & 1 \end{vmatrix} = +1(-1)^3 \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = \underline{\underline{-5}} \quad \checkmark$$

$$\textcircled{4} \left( \begin{array}{ccc|ccc} 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left( \begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right)$$

singularní matice,  $A^{-1}$  neexistuje ✓

$$\textcircled{5} \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 0 & -1 & -4 \\ 0 & 1 & -1 & 1 & 5 \end{array} \right) \sim \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & -3 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 1 & -1 & 1 & 5 \end{array} \right) \sim$$

$$\sim \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & -3 \\ 0 & 1 & -1 & 1 & 5 \\ 0 & 0 & -1 & -2 & -1 \end{array} \right)$$

$$x_1 + x_2 + x_3 + x_4 = -3$$

$$x_2 - x_3 + x_4 = 5$$

$$-x_3 - 2x_4 = -1$$

$$\underline{x_4 = r}$$

$$-x_3 - 2r = -1$$

$$-x_3 = 2r - 1$$

$$\underline{x_3 = 1 - 2r}$$

$$x_2 - (1 - 2r) + r = 5$$

$$x_2 - 1 + 3r = 5$$

$$\underline{x_2 = 6 - 3r}$$

$$x_1 + (6 - 3r) + (1 - 2r) + r = -3$$

$$x_1 + 7 - 4r = -3$$

$$\underline{x_1 = 4r - 10}$$

$$\underline{(4r - 10, 6 - 3r, 1 - 2r, r), r \in \mathbb{R}}$$