

1. S jakou pravděpodobností padne na dvou kostkách součet

- a) 6,  
b) menší než 10?

2. Jsou dány cifry 1, 2, 3, 4, 5. Kolik 4-ciferných čísel, v nichž se mohou cifry opakovat, lze z těchto cifer sestavit, chceme-li získat čísla končící cifrou 3?

3. Vypočítejte inverzní matici  $A^{-1}$  k matici  $A$  a proveďte zkoušku:  $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ .

4. Vypočítejte determinant

$$\begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 3 \\ 20 & 1 & 2 & 1 & 1 \end{vmatrix}$$

5. Vyřešte soustavu lineárních rovnic

$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 10 \\ x_1 + 3x_2 + x_3 &= 5 \\ 2x_1 + 2x_2 - x_3 &= 4 \end{aligned}$$

①

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(6) = \frac{5}{36}$$

$$P(<10) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

②  $X \times X \times 3 \quad 5 \cdot 5 \cdot 5 \cdot 1 = 5^3 = 125$

③  $(A|E_3) = \left( \begin{array}{ccc|ccc} 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array} \right) \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array} \right) \sim$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & +1 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array} \right) = (E_3 | A^{-1}) \quad A^{-1} = \frac{1}{2} \begin{pmatrix} 2 & 2 & -2 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

Zk:  $A^{-1} \cdot A = \frac{1}{2} \begin{pmatrix} 2 & 2 & -2 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E_3 \quad \text{ok.}$

$$\textcircled{3} \textcircled{4} \begin{pmatrix} 0 & \textcircled{1} & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 3 \\ 20 & 1 & 2 & 1 & 1 \end{pmatrix} = 1 \cdot (-1)^{1+2} \begin{pmatrix} 0 & 5 & 0 & 1 \\ \textcircled{1} & 0 & 0 & 0 \\ 10 & 1 & 2 & 3 \\ 20 & 2 & 1 & 1 \end{pmatrix} = \underbrace{(-1) \cdot 1 \cdot (-1)}_1 \begin{pmatrix} 5 & 0 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} =$$

$$= 5 \cdot (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 5 \cdot (2-3) + 1 \cdot (1-4) = -5-3 = \underline{\underline{-8}}$$

$$\textcircled{5} \begin{pmatrix} 1 & 2 & 2 & | & 10 \\ 1 & 3 & 1 & | & 5 \\ 2 & 2 & -1 & | & 4 \end{pmatrix} \underset{\substack{2.\text{ř.} - 1.\text{ř.} \\ 3.\text{ř.} - 2 \cdot 1.\text{ř.}}}{\sim} \begin{pmatrix} 1 & 2 & 2 & | & 10 \\ 0 & 1 & -1 & | & -5 \\ 0 & -2 & -5 & | & -16 \end{pmatrix} \underset{3.\text{ř.} + 2 \cdot 2.\text{ř.}}{\sim} \begin{pmatrix} 1 & 2 & 2 & | & 10 \\ 0 & 1 & -1 & | & -5 \\ 0 & 0 & -7 & | & -26 \end{pmatrix}$$

právě jedno řešení

$$x_1 + 2x_2 + 2x_3 = 10$$

$$x_2 - x_3 = -5$$

$$-7x_3 = -26 \Rightarrow x_3 = \frac{26}{7}$$

$$x_2 - \frac{26}{7} = -5$$

$$x_2 = -5 + \frac{26}{7} = \frac{-35+26}{7} = -\frac{9}{7}$$

$$x_1 + 2\left(-\frac{9}{7}\right) + 2\left(\frac{26}{7}\right) = 10$$

$$x_1 = 10 + \frac{18}{7} - \frac{52}{7}$$

$$x_1 = \frac{70+18-52}{7} = \frac{36}{7}$$

Jediné řešení  $\left(\frac{36}{7}, -\frac{9}{7}, \frac{26}{7}\right)$