Písemka	P1-2012-D

Dne: 9. listopadu 2012

KMA-MAT1

VZOR ....

1. Hod dvěma kostkami, bílou a černou:

   - Jev A: na bílé kostce padne číslo ≥ 2;
   - Jev B: na černé kostce padne číslo ≤ 4.

   $P(A) =?$, $P(B) =?$, $P(A \cap B) =?$, $P(A \cup B) =?$, $P(A/B) =?$ a $P(B/A) =?$.

2. Vypočtěte pravděpodobnost uhádnutí právě dvou čísel při tažení pěti čísel z dvaceti.

3. Vypočítejte inverzní matice A^{-1} a B^{-1} k maticím A a B a proveďte zkoušku:

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -3 & 1 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 2 & -2 & 0 \end{pmatrix}.$$

4. Vypočtěte determinant $\begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 2 \\ 1 & 11 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 2 \\ 15 & 1 & 2 & 1 & 3 \end{vmatrix}$.

5. Vyřešte soustavu lineárních rovnic
$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 5 \\ 3x_1 + 2x_2 + x_3 &= 6 \\ x_1 + x_2 - x_3 &= 4 . \end{aligned}$$

Řešení: (B, Č)

1)

			B		
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

A

$$P(A) = \frac{5 \cdot 6}{36} = \frac{30}{36} = \frac{5}{6}$$

$$P(B) = \frac{6 \cdot 4}{36} = \frac{24}{36} = \frac{2}{3}$$

$$P(A \cap B) = \frac{4 \cdot 5}{36} = \frac{20}{36} = \frac{5}{9}$$

$$P(A \cup B) = \frac{36-2}{36} = \frac{34}{36} = \frac{17}{18}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{9}}{\frac{2}{3}} = \frac{5}{9} \cdot \frac{3}{2} = \frac{5}{6}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{9}}{\frac{5}{6}} = \frac{5}{9} \cdot \frac{6}{5} = \frac{2}{3}$$

2)

$$P = \frac{\binom{5}{2} \cdot \binom{15}{3}}{\binom{20}{5}} = \frac{\frac{5!}{2!\,3!} \cdot \frac{15!}{3!\,12!}}{\frac{20!}{5!\,15!}} = \frac{\frac{5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{2} \cdot \cancel{3!}} \cdot \frac{\cancel{15}^{5} \cdot \cancel{14}^{7} \cdot 13 \cdot \cancel{12!}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{12!}}}{\frac{\cancel{20} \cdot 19 \cdot \cancel{18}_{3} \cdot 17 \cdot 16 \cdot \cancel{15!}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2} \cdot \cancel{15!}}} =$$

$$= \frac{5 \cdot \cancel{2} \cdot 5 \cdot 7 \cdot 13}{19 \cdot 3 \cdot 17 \cdot \cancel{16}\, 8} = \frac{2275}{7752} = 0{,}29347$$

③ $(A|E_3) = \left(\begin{array}{ccc|ccc} 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ -3 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ -3 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 3 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 5 & -1 & 3 & 1 \end{array}\right) \sim$

3.ř. + 3·1.ř. 3.ř. − 2.ř. $\frac{1}{5}$ 3.ř.

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/5 & 3/5 & 1/5 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/5 & -1/5 & -2/5 \\ 0 & 1 & 0 & 4/5 & 3/5 & 1/5 \\ 0 & 0 & 1 & -1/5 & 3/5 & 1/5 \end{array}\right) = (E_3 | A^{-1}) \quad A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -1 & -2 \\ 4 & 3 & 1 \\ -1 & 3 & 1 \end{pmatrix}$

1.ř. − 2·3.ř.
2.ř. + 3.ř.

Zkouška: $A^{-1} \cdot A = \frac{1}{5}\begin{pmatrix} 2 & -1 & -2 \\ 4 & 3 & 1 \\ -1 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -3 & 1 & -2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E_3$ OK

~~④~~ $(B|E_3) = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 2 & -2 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & -2 & -2 & -2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right)$

3.ř. − 2·1.ř. 3.ř. + 2.ř.

→ singulární matice => B^{-1} neexistuje

④ $\begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 2 \\ 1 & 1 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 2 \\ 15 & 1 & 2 & 1 & 3 \end{vmatrix} = 2 \cdot (-1)^{1+2} \begin{vmatrix} 0 & 3 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 10 & 1 & 2 & 2 \\ 15 & 2 & 1 & 3 \end{vmatrix} = -2 \cdot 1 \cdot (-1)^{2+1} \begin{vmatrix} 3 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 2 \cdot \left(3 \cdot (-1)^{1+1} \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} + 2 \cdot (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}\right) =$

$= 2\left(3(6-2) + 2(1-4)\right) = 2 \cdot (12-6) = 12$

⑤ $\left(\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 3 & 2 & 1 & 6 \\ 1 & 1 & -1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 3 & 2 & 1 & 6 \\ 2 & 1 & 3 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -1 & 4 & -6 \\ 0 & -1 & 5 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -1 & 4 & -6 \\ 0 & 0 & 1 & +3 \end{array}\right)$

2.ř. − 3·1.ř. 3.ř. − 2.ř.
3.ř. − 2·1.ř.

~~$x_1 + x_2 - x_3 = 4$~~
~~$-x_2 + 4x_3 = -6$~~
~~$x_3 = +3$~~

~~$x_3 = +3$~~
~~$-x_2 + 4(+3) = -6$~~
~~$-x_2 + 12 = -6$~~
~~$-x_2 = -2$~~
~~$x_2 = 2$~~

~~$-x_2 = -18$~~
~~$x_2 = 18$~~

~~$x_1 + 2 - (-1) = 4$~~
~~$x_1 + 3 = 4$~~
~~$x_1 = 1$~~

~~$x_1 + 18 - 3 = 4$~~
~~$x_1 + 15 = 4$~~
~~$x_1 = -11$~~

$x_1 + x_2 - x_3 = 4$
$-x_2 + 4x_3 = -6$
$x_3 = 3 \Rightarrow x_3 = 3$

$-x_2 + 4 \cdot 3 = -6$
$-x_2 = -18$
$x_2 = 18$

$x_1 + 18 - 3 = 4$
$x_1 + 15 = 4$
$x_1 = -11$

Řešení: $(-11, 18, 3)$