Písemka **P1-2012-F**

Dne: 23. listopadu 2012

KMA-MT1/MAT1

VZOR 1

1. Při hodu (jednou) hrací kostkou značí jev A „padnutí čísla ≤ 4“, jev B „padnutí čísla ≥ 3“. Určete následující pravděpodobnosti: $P(A) = ?$, $P(B) = ?$, $P(A \cap B) = ?$, $P(A \cup B) = ?$, $P(A/B) = ?$ a $P(B/A) = ?$.

2. Jsou dány cifry $1, 2, 3, 4, 5$. Kolik 5-ciferných čísel, v nichž se nemohou cifry opakovat, lze z těchto cifer sestavit, chceme-li získat čísla lichá?

3. Vypočítejte inverzní matici A^{-1} k matici A a proveďte zkoušku: $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 0 & 2 & 1 \end{pmatrix}$.

4. Vypočtěte determinant $\begin{vmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 3 \\ 10 & 1 & 2 & 1 & 2 \end{vmatrix}$.

5. Vyřešte soustavu lineárních rovnic
$$\begin{aligned} x_1 - 2x_2 - x_3 + x_4 &= 10 \\ x_1 + 3x_2 + x_3 - x_4 &= 6 \\ 2x_1 + x_2 - x_3 + x_4 &= 4 . \end{aligned}$$

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(1)

$A: \{1,2,3,4\} \quad P(A) = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

$B: \{3,4,5,6\} \quad P(B) = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

$A \cap B: \{3,4\} \quad P(A \cap B) = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$

$A \cup B: \{1,2,3,4,5,6\} \quad P(A \cup B) = \frac{6}{6} = \underline{\underline{1}}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \underline{\underline{\frac{1}{2}}}$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \underline{\underline{\frac{1}{2}}}$

(2)

$$\left.\begin{aligned} XXXX1 \quad 4! &= 24 \\ XXXX3 \quad 4! &= 24 \\ XXXX5 \quad 4! &= 24 \end{aligned}\right\} \; 3 \cdot 24 = \underline{\underline{72}}$$

(3)

$$(A|E_3) = \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & 3/2 & 1 \\ 0 & 1 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) = (E_3|A^{-1}) \qquad A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 3 & 2 \\ 1 & -1 & 0 \\ -2 & 2 & 2 \end{pmatrix}$$

③ - pokr. Zk $A^{-1} \cdot A = \frac{1}{2}\begin{pmatrix} -1 & 3 & 2 \\ 1 & -1 & 0 \\ -2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 0 & 2 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ok

④ $\begin{vmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 10 & 15 & 1 & 2 & 3 \\ 10 & 1 & 2 & 1 & 2 \end{vmatrix} = 3 \cdot (-1)^{1+2} \begin{vmatrix} 0 & 5 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 10 & 1 & 2 & 3 \\ 10 & 2 & 1 & 2 \end{vmatrix} = -3 \cdot 1 \cdot (-1)^{2+1} \begin{vmatrix} 5 & 0 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = 3 \cdot \left(5 \cdot (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} + \right.$

$\left. + 1 \cdot (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}\right) = 3\left(5 \cdot (4-3) + (1-4)\right) = \underline{\underline{6}}$

⑤ $\left(\begin{array}{cccc|c} 1 & -2 & -1 & 1 & 10 \\ 1 & 3 & 1 & -1 & 6 \\ 2 & 1 & -1 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -1 & 1 & 10 \\ 0 & 5 & 2 & -2 & -4 \\ 0 & 5 & 1 & -1 & -16 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -1 & 1 & 10 \\ 0 & 5 & 2 & -2 & -4 \\ 0 & 0 & -1 & 1 & -12 \end{array}\right)$

2.ř. − 1.ř.
3.ř. − 2·1.ř.

3.ř. − 2.ř.

$x_1 - 2x_2 - x_3 + x_4 = 10$

$5x_2 + 2x_3 - 2x_4 = -4$

$-x_3 + x_4 = -12$

Volíme: $\underline{x_4 = r}$

$-x_3 + r = -12$

$-x_3 = -r - 12$

$\underline{x_3 = r + 12}$

$5x_2 + 2(r+12) - 2r = -4$

$5x_2 + 2r + 24 - 2r = -4$

$5x_2 = -28$

$\underline{x_2 = -\frac{28}{5}}$

$x_1 - 2 \cdot \left(-\frac{28}{5}\right) - (r+12) + r = 10$

$x_1 + \frac{56}{5} - r - 12 + r = 10$

$x_1 = 10 + 12 - \frac{56}{5}$

$\underline{x_1} = \frac{50+60-56}{5} = \underline{\frac{54}{5}}$

Řešení:

~~$\left(\frac{54}{5}, -\frac{28}{5}, r+12, r\right), r \in \mathbb{R}$~~

$\underline{\underline{\left(\frac{54}{5}, -\frac{28}{5}, r+12, r\right), r \in \mathbb{R}}}$