(4)
$$\begin{vmatrix} 0 & \boxed{-2} & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 5 & 15 & 1 & 2 & -2 \\ 10 & 100 & 2 & 1 & -2 \end{vmatrix} = \underbrace{(-2)\cdot(-1)^{1+2}}_{2} \begin{vmatrix} 0 & 5 & 0 & 1 \\ \boxed{1} & 0 & 0 & 0 \\ 5 & 1 & 2 & -2 \\ 10 & 2 & 1 & -2 \end{vmatrix} = \underbrace{2\cdot 1\cdot(-1)^{2+1}}_{-2} \begin{vmatrix} 5 & 0 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & -2 \end{vmatrix} =$$

$$= -2\left[5\cdot(-1)^{1+1}\begin{vmatrix} 2 & -2 \\ 1 & -2 \end{vmatrix} + 1\cdot(-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}\right] = -2\left[5(-4+2)+1\cdot(1-4)\right] =$$

$$= -2\left[-10-3\right] = \underline{\underline{26}}$$

(5)
$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & -2 & 2 \\ 1 & 3 & -1 & -2 & 1 \\ 2 & 1 & -1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & -2 & 2 \\ 0 & 2 & 0 & 0 & -1 \\ 0 & -1 & 1 & 5 & -5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & -3 \\ 0 & 2 & 0 & 0 & -1 \\ 0 & -1 & 1 & 5 & -5 \end{array}\right)$$

2.ř. − 1.ř.
3.ř. − 2 · 1.ř.

1.ř. + 3.ř.

$$x_1 + 3x_4 = -3$$
$$2x_2 = -1 \longrightarrow \underline{x_2 = \frac{-1}{2}}$$
$$\underline{-x_2 + x_3 + 5x_4 = -5}$$

$\underline{x_4 = r}$ − volíme

$$x_1 + 3r = -3$$
$$\underline{x_1 = -3 - 3r}$$

$$-\left(\frac{-1}{2}\right) + x_3 + 5\cdot r = -5$$
$$x_3 = -5 - 5r - \frac{1}{2}$$
$$x_3 = -5r - 5{,}5$$
$$\underline{x_3 = \frac{1}{2}(-10r - 11)}$$

řešení: $\underline{\underline{\left(-3-3r,\ -\frac{1}{2},\ \frac{1}{2}(-10r-11),\ r\right)}}$, $r \in \mathbb{R}$

Písemka **P1-2012-G**	Dne: 23. listopadu 2012 KMA-MT1/MAT1

VZOR

1. Při hodu (jednou) hrací kostkou značí jev A „padnutí čísla ≥ 4", jev B „padnutí čísla ≥ 3". Určete následující pravděpodobnosti: $P(A) =?$, $P(B) =?$, $P(A \cap B) =?$, $P(A \cup B) =?$, $P(A/B) =?$ a $P(B/A) =?$.

2. Jsou dány cifry $1, 2, 3, 4, 5$. Kolik 5-ciferných čísel, v nichž se nemohou cifry opakovat, lze z těchto cifer sestavit, chceme-li získat čísla sudá?

3. Vypočítejte inverzní matici A^{-1} k matici A a proveďte zkoušku: $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$.

4. Vypočtěte determinant $\begin{vmatrix} 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 \\ 1 & 6 & 0 & 0 & 0 \\ 5 & 15 & 1 & 2 & -2 \\ 10 & 100 & 2 & 1 & -2 \end{vmatrix}$.

5. Vyřešte soustavu lineárních rovnic
$$\begin{aligned} x_1 + x_2 - x_3 - 2x_4 &= 2 \\ x_1 + 3x_2 - x_3 - 2x_4 &= 1 \\ 2x_1 + x_2 - x_3 + x_4 &= -1 \end{aligned}$$

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(1) $1 \; 2 \; \underbrace{3 \; \overbrace{4 \; 5 \; 6}^{A}}_{B}$

$P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{4}{6} = \frac{2}{3}$

$A \cup B = B$; $P(A \cup B) = \frac{2}{3}$

$A \cap B = A$; $P(A \cap B) = 1/2$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

(2) XXXX2 $\quad 4! = 24$

XXXX4 $\quad 4! = 24$

48 možností

(3) $$(A|E_3) = \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & -2 & 0 & 1/2 & 1 & -3/2 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & -1/4 & -1/2 & 3/4 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & 1/2 & -1/4 \\ 0 & 1 & 0 & -1/4 & -1/2 & 3/4 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right) = (E_3|A^{-1})$$

$$A^{-1} = \begin{pmatrix} 3/4 & 1/2 & -1/4 \\ -1/4 & -1/2 & 3/4 \\ -1/2 & 0 & 1/2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & 2 & -1 \\ -1 & -2 & 3 \\ -2 & 0 & 2 \end{pmatrix},$$

Zk.: $$A^{-1} \cdot A = \frac{1}{4}\begin{pmatrix} 3 & 2 & -1 \\ -1 & -2 & 3 \\ -2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Ok