UKÁZKA DRUHÉ ZÁPOČTOVÉ PÍSEMKY

Písemka LS-2013	P2–E

~~27. května 2013~~

KMA-MAT/MT/2 VZOR ....

1. Vyřešte diferenciální rovnici $y' = t - y$.
2. Určete lokální extrémy funkce $f(x, y) = x^2y^2 - x^2 - y^2 + 30$.
3. Pro uzly x_i a funkční hodnoty f_i dané tabulkou

i	0	1	2
x_i	-1	0	1
f_i	1	-1	1

sestavte interpolační polynom: a) v základním tvaru, b) v Lagrangeově tvaru.

① $y' = t - y \quad \to \quad y' + y = t$ lineární DR 1. ř. $(t \in \mathbb{R}, y \in \mathbb{R})$

HÚ: $y' + y = 0$, OŘHÚ: $y = C \cdot e^{-\int 1\,dt} = C \cdot e^{-t}$

NHÚ: Variace konstanty

$$y = K(t) \cdot e^{-t}, \quad y' = K'(t) \cdot e^{-t} - K(x) e^{-t}$$

dosadíme do NHÚ:

$$K'(t) \cdot e^{-t} \underbrace{- K(t) \cdot e^{-t} + K(t) \cdot e^{-t}} = t$$

$$K'(t) \cdot e^{-t} = t, \quad K'(t) = t \cdot e^{t},$$

$$K(t) = \int t \cdot e^{t}\,dt = \begin{bmatrix} u = t & v' = e^{t} \\ u' = 1 & v = e^{t} \end{bmatrix} = t \cdot e^{t} - \int e^{t}\,dt = t \cdot e^{t} - e^{t} + C$$

OŘNHÚ:

$$y = K(t) \cdot e^{-t} = (t \cdot e^{t} - e^{t} + C) \cdot e^{-t} = t - 1 + C \cdot e^{-t}$$

$$\boxed{y = t - 1 + C \cdot e^{-t}, \quad C \in \mathbb{R}, \ t \in \mathbb{R}}$$

② Stacionární body: $\dfrac{\partial f}{\partial x} = 0$, $\dfrac{\partial f}{\partial y} = 0$ $\qquad 2xy^2 - 2x = 0$, $\quad 2x^2y - 2y = 0$

$x = 0 \qquad y = \pm 1$

$2x(y^2 - 1) = 0$

$2y(x^2 - 1) = 0$

$x = 0:\ 2y(0 - 1) = 0,\ -2y = 0,\ y = 0$

$y = 1:\ 2 \cdot 1(x^2 - 1) = 0,\ 2x^2 = 2,\ x^2 = 1,\ x = \pm 1$

$y = -1:\ 2 \cdot (-1)(x^2 - 1) = 0,\ x^2 = 1,\ x = \pm 1$

$A = [0, 0]$, $B = [-1, 1]$, $C = [1, 1]$, $D = [-1, -1]$, $E = [+1, -1]$

$$D_1(x,y) = \frac{\partial^2 f}{\partial x^2}(x,y) = 2y^2 - 2 = 2(y^2-1)$$

$$D_2(x,y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(x,y) & \frac{\partial^2 f}{\partial x \partial y}(x,y) \\ \frac{\partial^2 f}{\partial y \partial x}(x,y) & \frac{\partial^2 f}{\partial y^2}(x,y) \end{vmatrix} = \begin{vmatrix} 2(y^2-1) & 4xy \\ 4xy & 2(x^2-1) \end{vmatrix} = 4(y^2-1)(x^2-1) - 16x^2y^2$$

$A = [0,0]$

$D_2(0,0) = 4(-1)(-1) - 16 \cdot 0 = 4 > 0$, $D_1(0,0) = -2 \cancel{>} < 0 \Rightarrow$ ostvé lokální ~~minimum~~ maximum $f(0,0) = 30$

$B = [-1,1]$

$D_2(-1,1) = 4 \cdot 0 \cdot 0 - 16 \cdot 1 \cdot 1 = -16 \rightarrow$ nemá LExtr.

$C = [1,1]$

$D_2(1,1) = 4 \cdot 0 - 16 < 0 \rightarrow$ nemá LExtr.

Pro D a E to platí také.

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i	0	1	2
x_i	-1	0	1
f_i	1	-1	1

$n = 2 \Rightarrow p_n(x) = p_2(x) = a_0 + a_1 x + a_2 x^2$

a) $p_2(-1) = 1$ $\quad a_0 - a_1 + a_2 = 1$

$p_2(0) = -1$ $\quad a_0 = -1 \Rightarrow \boxed{a_0 = -1}$

$p_2(1) = 1$ $\quad a_0 + a_1 + a_2 = 1$

$-1 - a_1 + a_2 = 1$

$-1 + a_1 + a_2 = 1$

$-2 + 2a_2 = 2$

$2a_2 = 4$

$\boxed{a_2 = 2}$

$-1 - a_1 + 2 = 1$

$\boxed{a_1 = 0}$

$p_2(x) = -1 + 0 \cdot x + 2x^2$

$p_2(x) = 2x^2 - 1$

b) $p_2(x) = f_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

$p_2(x) = 1 \cdot \frac{(x)(x-1)}{(-1)(-1-1)} - 1 \cdot \frac{(x+1)(x-1)}{(0+1)(0-1)} + 1 \cdot \frac{(x+1)(x)}{(1+1)(1)}$

$p_2(x) = \frac{1}{2} x(x-1) + 1(x+1)(x-1) + \frac{1}{2}(x+1) \cdot x$

$p_2(x) = \frac{1}{2}x^2 - \frac{1}{2}x + x^2 - 1 + \frac{1}{2}x^2 + \frac{1}{2}x$

$p_2(x) = 2x^2 - 1$