Písemka LS-2011 P2–𝔽

19. května 2011

KMA-MAT2/MT2 ....

PÍSEMKA JE ZAMÝŠLENA BEZ „TAHÁKU“!

1. $$\int \frac{1-\sqrt[3]{\operatorname{cotg} x}}{\sin^2 x}\,dx = \left[\begin{array}{l} t=\operatorname{cotg} x \\ dt = \frac{-dx}{\sin^2 x}\end{array}\right] = -\int\left(1-\sqrt[3]{t}\right)dt = \int\left(\sqrt[3]{t}-1\right)dt =$$

$$= \int\left(t^{\frac{1}{3}}-1\right)dt = \frac{t^{\frac{4}{3}}}{\frac{4}{3}} - t + C = \underline{\frac{3}{4}\sqrt[3]{(\operatorname{cotg} x)^4} - \operatorname{cotg} x + C}$$

2. $$\int_{-2}^{0} 2\,|x+1|\,dx = \left[\, 2|x+1| = \begin{cases} 2(-x-1) = -2x-2 & \text{pro } x < -1 \\ 2(x+1) = 2x+2 & \text{pro } x \geq -1 \end{cases}\right] =$$

$$= \int_{-2}^{-1}(-2x-2)\,dx + \int_{-1}^{0}(2x+2)\,dx = -\left[x^2+2x\right]_{x=-2}^{-1} + \left[x^2+2x\right]_{x=-1}^{0} =$$

$$= -\big((1-2)-(4-4)\big) + \big(0-(1-2)\big) = 1+1 = \underline{2}$$

3. $$\int_0^{2\pi} x \cdot \cos x\,dx = \left[x\sin x + \cos x\right]_{x=0}^{2\pi} = (2\pi\cdot 0 + 1) - (0\cdot 0 + 1) = \underline{0}$$

$$\int x\cdot\cos x\,dx = \left[\begin{array}{ll} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{array}\right] = x\cdot\sin x - \int \sin x\,dx =$$

$$= x\sin x + \cos x + C$$

4. $\int \left(\frac{2}{\sin^2 x} - 6 \cdot 9^x + \sqrt[6]{x^5} + 21\right) \mathrm{d}x = -2 \cdot \operatorname{cotg} x - 6 \cdot \frac{9^x}{\ln 9} + \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + 21x + C =$

$= -2 \operatorname{cotg} x - \frac{6}{\ln 9} \cdot 9^x + \frac{6}{11} \sqrt[6]{x^{11}} + 21x + C$

5. $\int \operatorname{tg} x \,\mathrm{d}x = \int \frac{\sin x}{\cos x} dx = \begin{bmatrix} t = \cos x \\ dt = -\sin x \, dx \end{bmatrix} = -\int \frac{dt}{t} = -\ln|t| + C =$

$= -\ln|\cos x| + C$

6. $\int \operatorname{arctg} x \cdot \frac{\mathrm{e}^{\operatorname{arctg} x}}{1+x^2} \mathrm{d}x = \begin{bmatrix} t = \operatorname{arctg} x \\ dt = \frac{dx}{1+x^2} \end{bmatrix} = \int t \cdot e^t dt = \begin{bmatrix} u = t & v' = e^t \\ u' = 1 & v = e^t \end{bmatrix} =$

$= t \cdot e^t - \int e^t dt = t \cdot e^t - e^t + C = (t-1)\, e^t + C =$

$= (\operatorname{arctg} x - 1) \cdot e^{\operatorname{arctg} x} + C$