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Písemka LS-2013 **P1–B**

27. března 2013

KMA-MAT2/MT2 ....

5 1. Pro funkci $f : y = \sqrt{3x^2 - 2x}\,\sin(x+5)$ určete $D(f)$, f' a $D(f')$.

2. Vypočtěte následující integrály. Nezapomeňte na určení příslušných intervalů.

4,5 (a) $\int_0^1 \sqrt{x}(1 + 2x^2)\,dx$,

5 (b) $\int (x^2 - 2x + 1)\sin x\,dx$,

5 (c) $\int \frac{x - \operatorname{arctg} x}{1 + x^2}\,dx$.

10 3. Graficky znázorněte a vypočtěte obsah konečného rovinného obrazce ohraničeného křivkami: $f : y = e^x$, $g : y = e^{-x}$ a $x = 1$.

① $D(f):\ 3x^2 - 2x \geq 0,\quad x(3x-2) \geq 0 \quad 0,\ \frac{2}{3}$

$D(f) = (-\infty; 0\rangle \cup \langle \frac{2}{3}, +\infty)$

$y' = \frac{1}{2\sqrt{3x^2-2x}} \cdot (6x-2) \cdot \sin(x+5) + \sqrt{3x^2-2x} \cdot \cos(x+5)$

$D(f'):\ (3x^2 - 2x \geq 0) \wedge ((3x^2-2x) \neq 0) \Rightarrow D(f') = (-\infty, 0) \cup (\frac{2}{3}; +\infty)$

②a. $\int \sqrt{x}(1+2x^2)\,dx = \int(\sqrt{x} + 2\cdot\sqrt{x}\cdot x^2)\,dx = \int(\sqrt{x} + 2x^{\frac{1}{2}+2})\,dx = \int(\sqrt{x} + 2x^{\frac{5}{2}})\,dx =$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C = \frac{2}{3}\sqrt{x^3} + \frac{4}{7}\sqrt{x^7} + C \qquad x \in \langle 0; +\infty)$

$\int_0^1 \sqrt{x}(1+2x^2)\,dx = \left[\frac{2}{3}\sqrt{x^3} + \frac{4}{7}\sqrt{x^7}\right]_0^1 = \left(\frac{2}{3} + \frac{4}{7}\right) - 0 = \frac{14+12}{21} = \frac{26}{21}$

②b $\int (x^2 - 2x + 1)\sin x\,dx = \left[\begin{matrix} u = x^2-2x+1 & v' = \sin x \\ u' = 2x-2 & v = -\cos x \end{matrix}\right] = (x^2-2x+1)(-\cos x) +$

$+ \int (2x-2)\cos x\,dx = \left[\begin{matrix} u = 2x-2 & v' = \cos x \\ u' = 2 & v = \sin x \end{matrix}\right] = -(x^2-2x+1)\cos x + (2x-2)\sin x -$

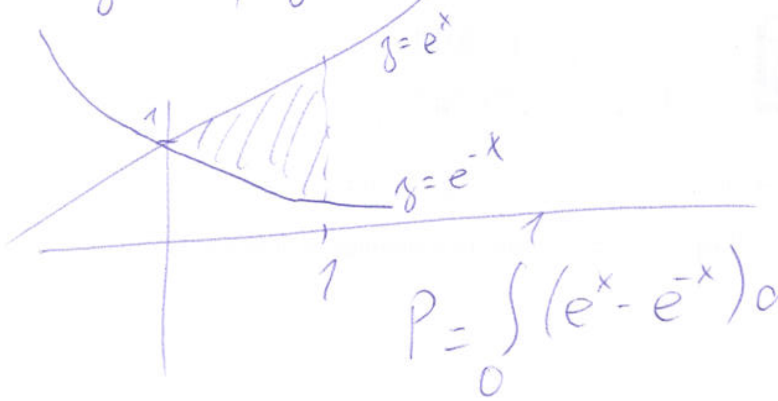
$- \int 2\sin x\,dx = -(x^2-2x+1)\cos x + (2x-2)\sin x + 2\cos x + C,\ x \in \mathbb{R}$

②c $\int \frac{x - \operatorname{arctg} x}{1+x^2}\,dx = \int \frac{x\,dx}{1+x^2} - \int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2) - \frac{\operatorname{arctg}^2 x}{2} + C \quad x \in \mathbb{R}$

$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + C,\quad \int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \left[\begin{matrix} t = \operatorname{arctg} x \\ dt = \frac{1}{1+x^2}\,dx \end{matrix}\right] = \int t\,dt = \frac{t^2}{2} + C =$

$= \frac{\operatorname{arctg}^2 x}{2} + C$

(3) $y = e^x$, $y = e^{-x}$, $x = 1$



$$P = \int_0^1 (e^x - e^{-x})\,dx = \left[e^x + e^{-x}\right]_0^1 = (e^1 + e^{-1}) - (e^0 + e^0) =$$

$$= e + \frac{1}{e} - 2$$