28/30

Písemka LS-2013 **P1-ℂ**

27. března 2013

KMA-MAT2/MT2

. . . .

3/5 1. Pro funkci $f : y = \sqrt{2x^2 + 3x}\,\sin(5 - x)$ určete $D(f)$, f' a $D(f')$.

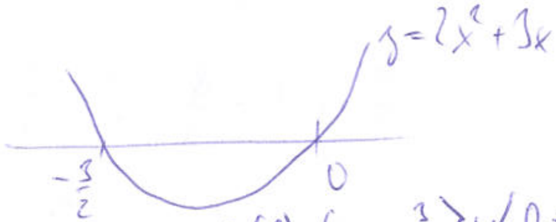
2. Vypočtěte následující integrály. Nezapomeňte na určení příslušných intervalů.

5 (a) $\int_0^1 \sqrt{x^3}(1 - x^2)\,dx$,

5 (b) $\int x^2 \sin x\,dx$,

5 (c) $\int_0^2 |x^2 - 1|\,dx$.

10 3. Graficky znázorněte a vypočtěte obsah konečného rovinného obrazce ohraničeného křivkami: $xy = 1$, $x = 1$, $x = 2$ a $y = 0$.



1) $D(f)$: $2x^2 + 3x \geq 0$, $x(2x+3) \geq 0$
0, $-\frac{3}{2}$

$D(f) = (-\infty, -\frac{3}{2}\rangle \cup \langle 0, +\infty)$

$$y' = \frac{1\cdot(4x+3)}{2\sqrt{2x^2+3x}} \sin(5-x) + \sqrt{2x^2+3x}\cdot\cos(5-x)\cdot(-1)$$

$$D(f') = (-\infty, -\tfrac{3}{2}) \cup (0, +\infty)$$

2a) $$\int \sqrt{x^3}(1-x^2)\,dx = \int x^{\frac{3}{2}}(1-x^2)\,dx = \int \left(x^{\frac{3}{2}} - x^{\frac{7}{2}}\right)dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{9}{2}}}{\frac{9}{2}} + C$$

$$\int_0^1 \sqrt{x^3}(1-x^2)\,dx = \left[\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{9}x^{\frac{9}{2}}\right]_0^1 = \frac{2}{5} - \frac{2}{9} = \frac{18-10}{45} = \frac{8}{45}$$ $x \in \langle 0, \infty)$

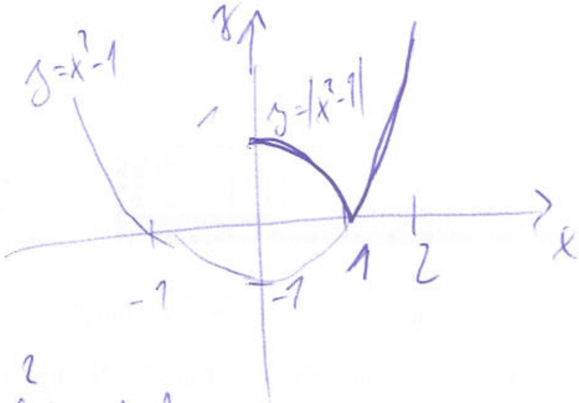
2b) $$\int x^2 \sin x\,dx = \begin{bmatrix} u = x^2 & v' = \sin x \\ u' = 2x & v = -\cos x \end{bmatrix} = -x^2\cos x + 2\int x\cos x\,dx =$$

$$= \begin{bmatrix} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{bmatrix} = -x^2\cos x + 2x\sin x - 2\int \sin x\,dx =$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$ $x \in \mathbb{R}$

2c)

2c) $\int_0^2 |x^2-1|\,dx$

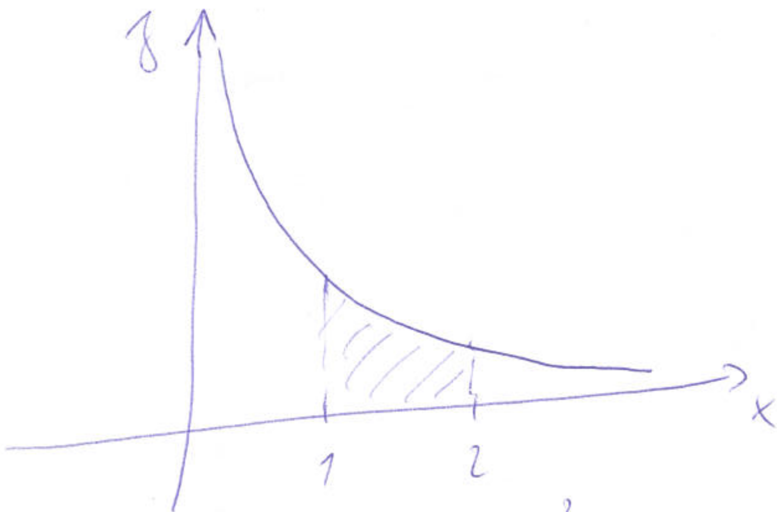


$$\int_0^2 |x^2-1|\,dx = \int_0^1 -(x^2-1)\,dx + \int_1^2 (x^2-1)\,dx =$$

$$= \left[-\frac{x^3}{3}+x\right]_0^1 + \left[\frac{x^3}{3}-x\right]_1^2 = \left(-\frac{1}{3}+1\right)-0+\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)=$$

$$= -\frac{1}{3}+1+\frac{8}{3}-2-\frac{1}{3}+1 = \frac{6}{3} = \underline{\underline{2}}\ [j^2]$$

3) $x\cdot y = 1 \Rightarrow y = \frac{1}{x}$, $x=1$, $x=2$, $y=0$



$$P = \int_1^2 \frac{1}{x}\,dx = \left[\ln|x|\right]_1^2 = \left[\ln x\right]_1^2 = \ln 2 - \ln 1 =$$

$$= \ln 2 - 0 = \underline{\ln 2\ [j^2]}$$