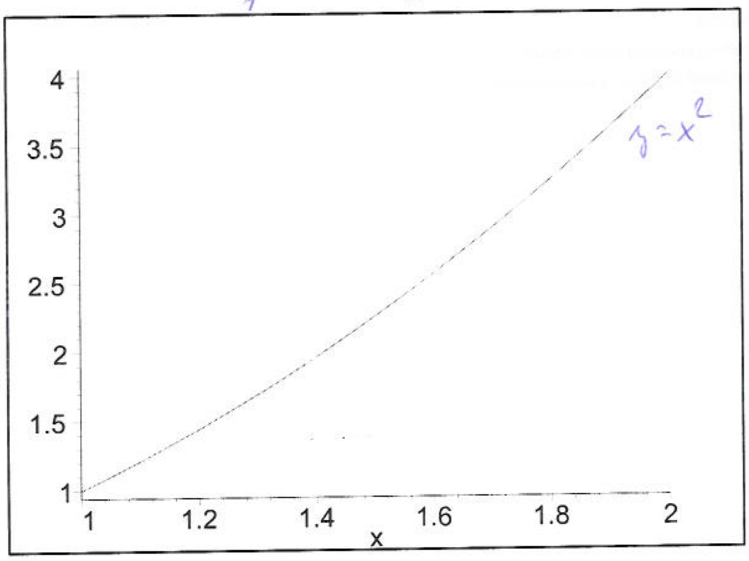
$$\int_1^2 x^2\,dx$$

```
> int(x^2,x);
> int(x^2,x=1..2);
> plot(x^2,x=1..2);
```

PF $\frac{1}{3}x^3$ na $\mathbb{R}$

$\int_1^2 x^2 dx = \frac{7}{3}$

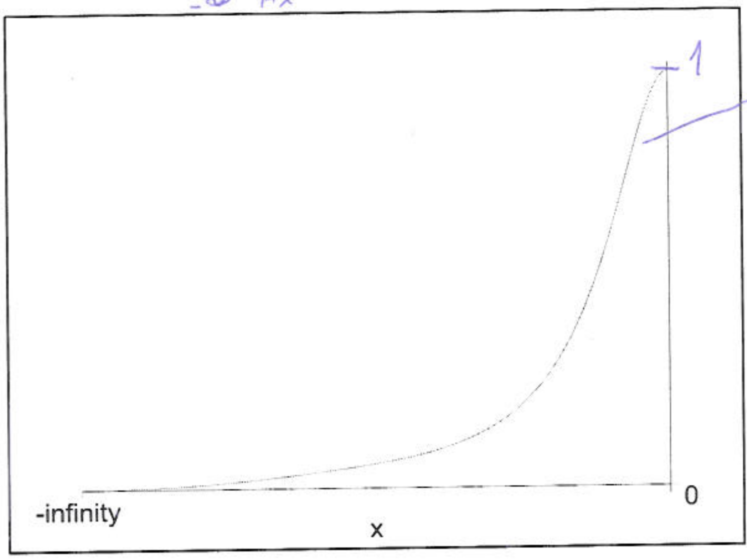


$$\int_{-\infty}^{0} \frac{dx}{1+x^2}$$

```
> int(1/(1+x^2),x);
> int(1/(1+x^2),x=-infinity..0);
> plot(1/(1+x^2),x=-infinity..0);
```

$$\arctan(x) \qquad \text{PF na } \mathbb{R}$$

$$\int_{-\infty}^{0} \frac{1}{1+x^2}\,dx = \frac{1}{2}\pi$$

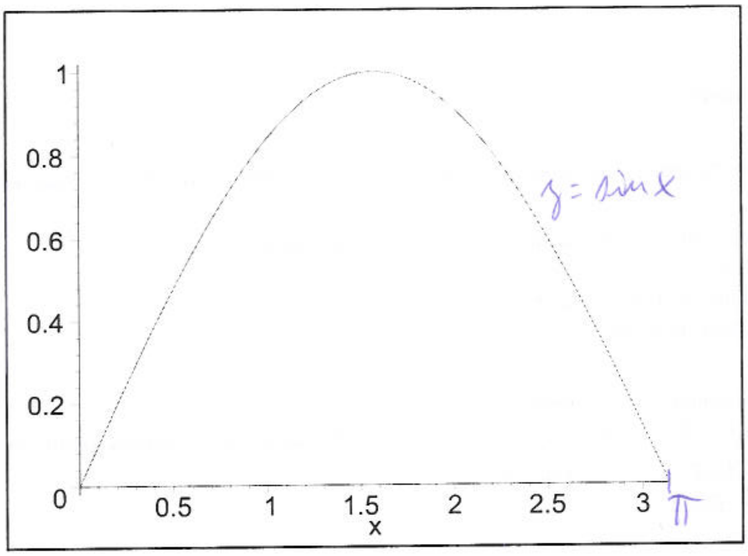


$$\int_0^{\pi} \sin x \, dx$$

```
> int(sin(x) ,x  );
> int(sin(x) ,x=0..Pi  );
> plot(sin(x) ,x=0..Pi  );
```

$$-\cos(x)$$ PF na $\mathbb{R}$

$$2$$ —— $\int_0^{\pi} \sin x \, dx$

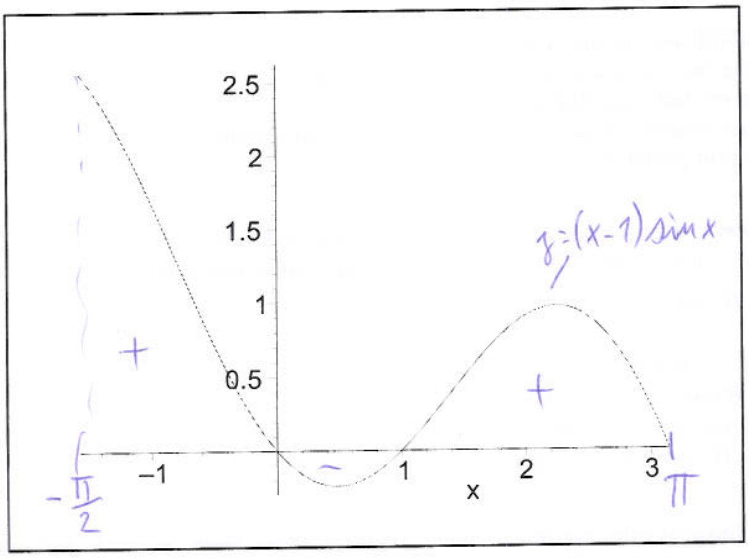


$$\int_{-\frac{\pi}{2}}^{\pi}$$

```
> int((x-1)*sin(x),x);
> int((x-1)*sin(x),x=-Pi/2..Pi);
> plot((x-1)*sin(x),x=-Pi/2..Pi);
```

$$\sin(x) - x\cos(x) + \cos(x)$$ — PF na $\mathbb{R}$

$$\pi$$ — $\int_{-\frac{\pi}{2}}^{\pi}(x-1)\sin x\,dx$

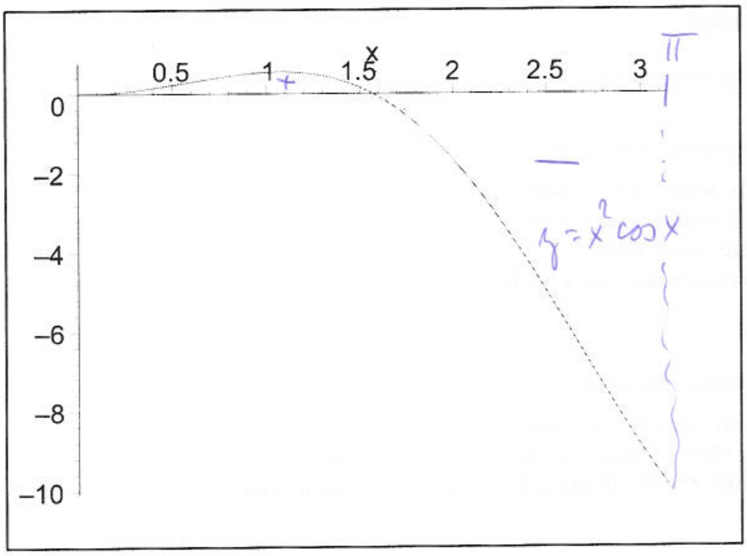


$$\int_0^{\pi} x^2 \cos x \, dx$$

```
> int(x^2*cos(x),x);
> int(x^2*cos(x),x=0..Pi);
> plot(x^2*cos(x),x=0..Pi);
```

$$x^2 \sin(x) - 2\sin(x) + 2\,x\cos(x)$$ PF на $\mathbb{R}$

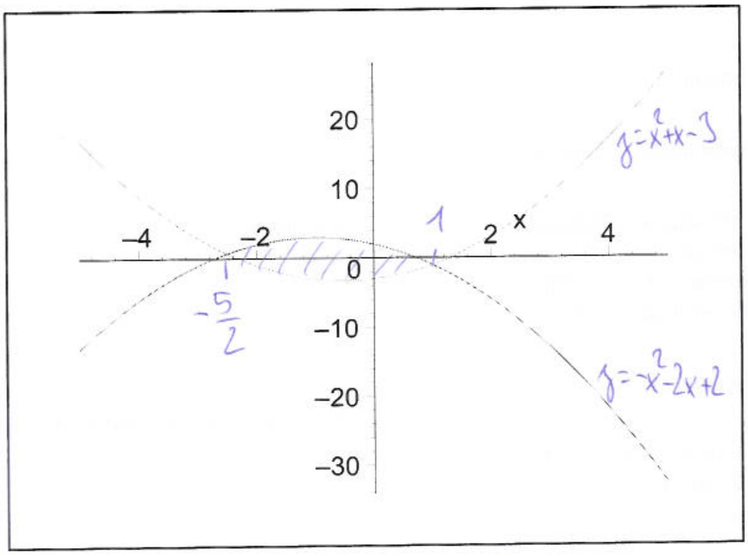
$$-2\,\pi$$ $\int_0^{\pi} x^2 \cos x \, dx$



1

```
> solve(x^2+x-3=-x^2-2*x+2,x);
> plot({x^2+x-3,-x^2-2*x+2},x=-5..5);
> int((-x^2-2*x+2)-(x^2+x-3),x);
> int((-x^2-2*x+2)-(x^2+x-3),x=-5/2..1);
```

$$1, \frac{-5}{2}$$ – průsečíky (x-ové souřadnice průsečíků)

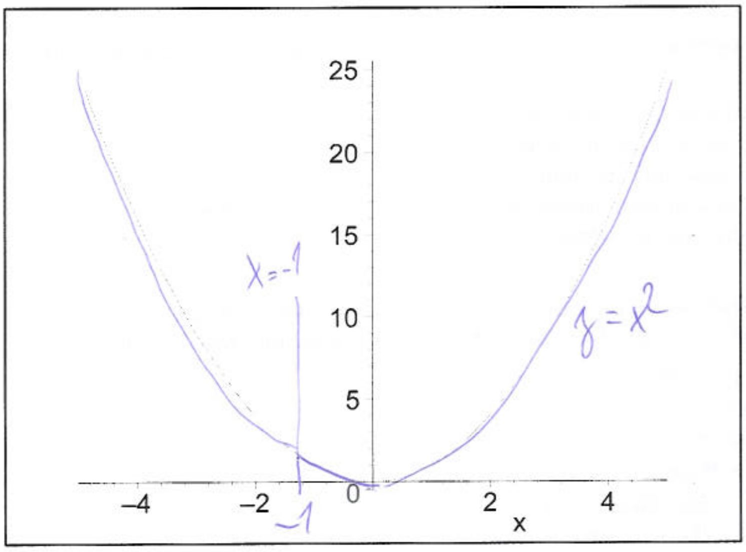


$$-\frac{2}{3}x^3-\frac{3}{2}x^2+5x$$ PF na $\mathbb{R}$

$$P = \frac{343}{24} = \int_{-\frac{5}{2}}^{1}\left[\left(-x^2-2x+2\right)-\left(x^2+x-3\right)\right]dx$$

2

```
> plot({0,x^2,x=-1},x=-5..5);
> int(x^2,x);
> int(x^2,x=-1..0);
```



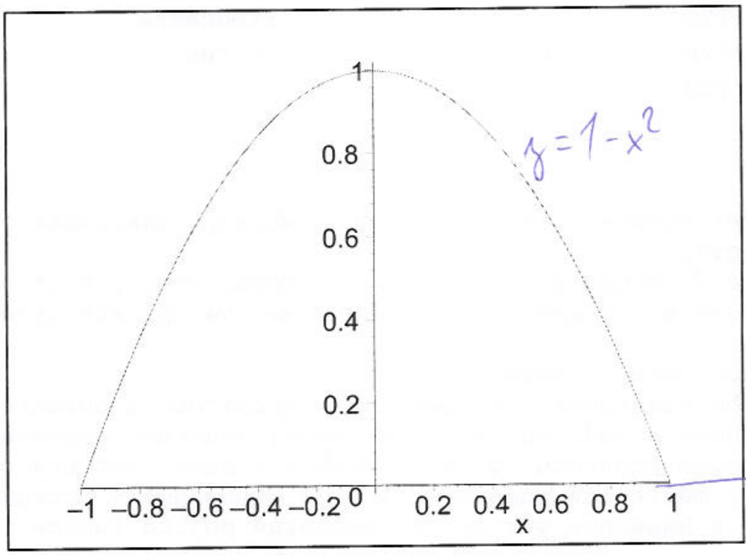
$\frac{1}{3}x^3$ / PF na $\mathbb{R}$

$\frac{1}{3} = \int_{-1}^{0} x^2 dx$

3

```
> solve(1-x^2=0,x);
> plot(1-x^2,x=-1..1);
> int(1-x^2,x);
> int(1-x^2,x=-1..1);
```

$-1, 1$ – nulové body



$y = 1 - x^2$

$y = 0$

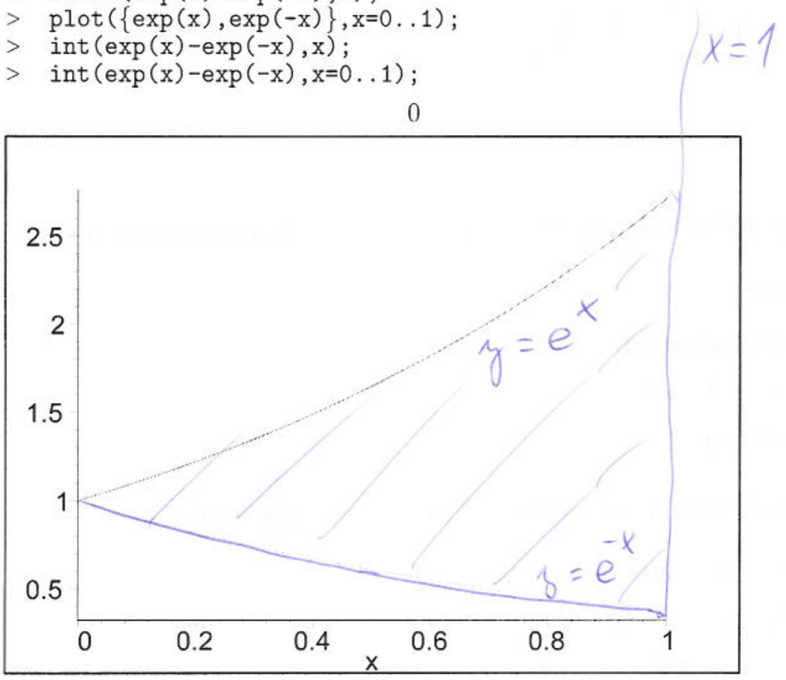
$x - \frac{1}{3}x^3$ PF na $\mathbb{R}$

$\frac{4}{3} = \int_{-1}^{1} (1 - x^2)\,dx$

4

```
> solve(exp(x)=exp(-x),x);
> plot({exp(x),exp(-x)},x=0..1);
> int(exp(x)-exp(-x),x);
> int(exp(x)-exp(-x),x=0..1);
```

0



PF na R

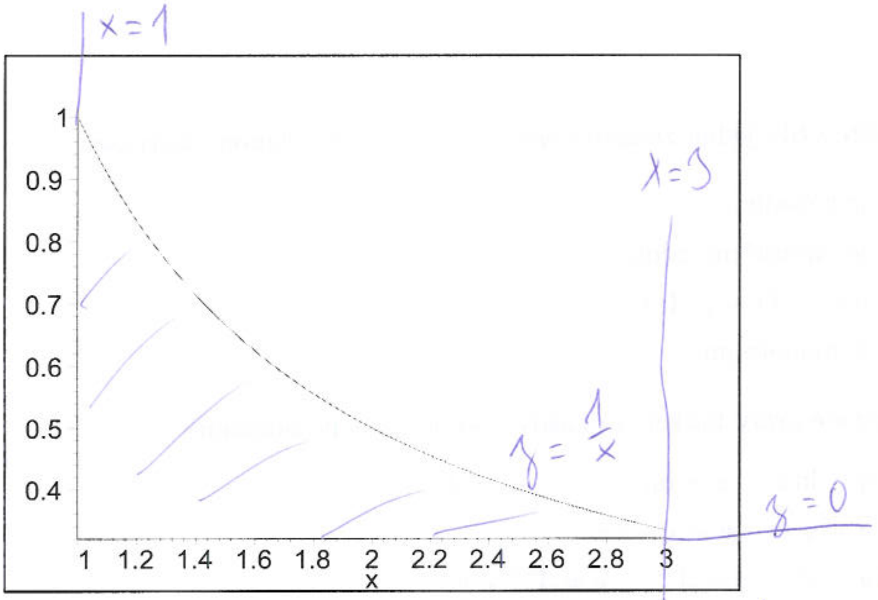
$$e^x + e^{(-x)}$$

$$e + e^{(-1)} - 2$$

$$= \int_0^1 (e^x - e^{-x})\,dx$$

5

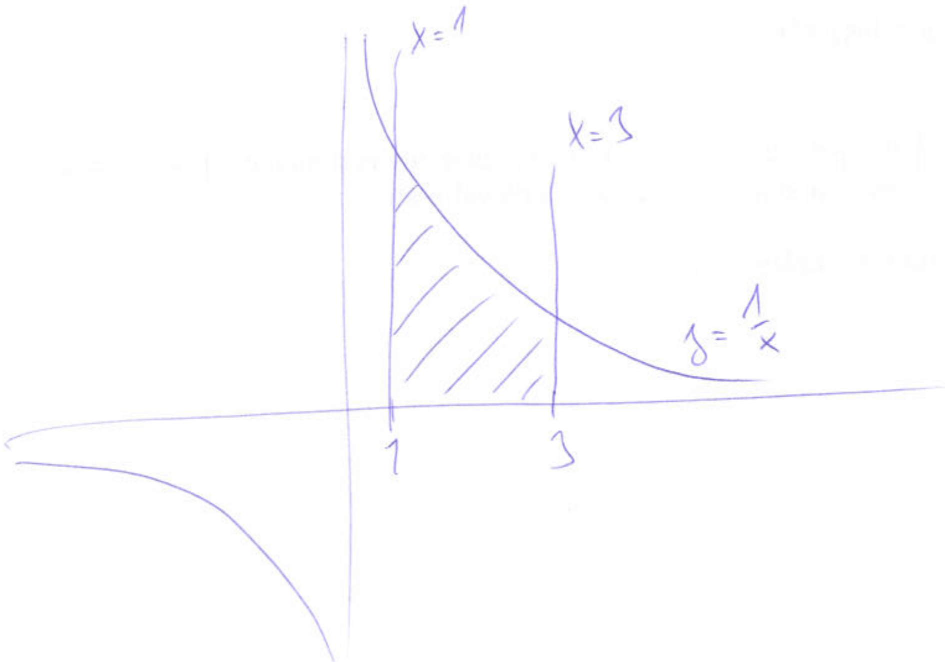
```
> plot(1/x,x=1..3);
> int(1/x,x);
> int(1/x,x=1..3);
```



$\ln(x)$ — PF na $(0,+\infty)$

$\ln(3) = \int_1^3 \frac{1}{x}\,dx$

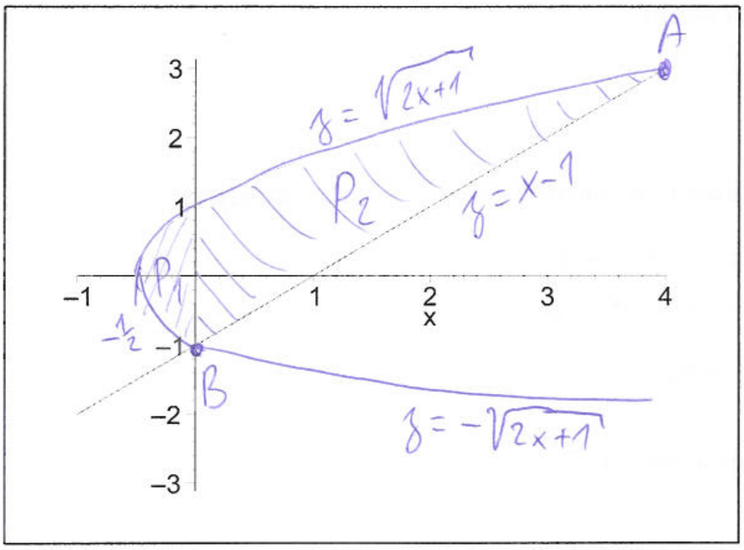
$x \cdot y = 1 \Rightarrow y = \frac{1}{x}$ $\quad (x \in \langle 1;3 \rangle \Rightarrow x \neq 0)$



6

```
> solve(sqrt(2*x+1)=x-1,x);
> solve(-sqrt(2*x+1)=x-1,x);
> plot({sqrt(2*x+1),-sqrt(2*x+1),x-1},x=-1..4);
> int(sqrt(2*x+1),x);
> int(sqrt(2*x+1)-(x-1),x);
> int(sqrt(2*x+1),x=-1/2..0);
> int(sqrt(2*x+1)-(x-1),x=0..4);
> int(sqrt(2*x+1),x=-1/2..0)+
> int(sqrt(2*x+1)-(x-1),x=0..4);
```

4
0 > průsečíky A, B



$y^2 = 2x+1$

$y = \pm\sqrt{2x+1}$

$x \geq -\frac{1}{2}$

$$\frac{1}{3}(2x+1)^{(3/2)} \quad \text{— PF na } \left(-\tfrac{1}{2}, +\infty\right)$$

$$\frac{1}{3}(2x+1)^{(3/2)} - \frac{1}{2}x^2 + x$$

$$\frac{1}{3} = \int_{-\frac{1}{2}}^{0} \left[\sqrt{2x+1} - \left(-\sqrt{2x+1}\right)\right] dx = P_1$$

$$\frac{14}{3} = \int_0^4 \left(\sqrt{2x+1} - (x-1)\right) dx = P_2$$

$$5$$

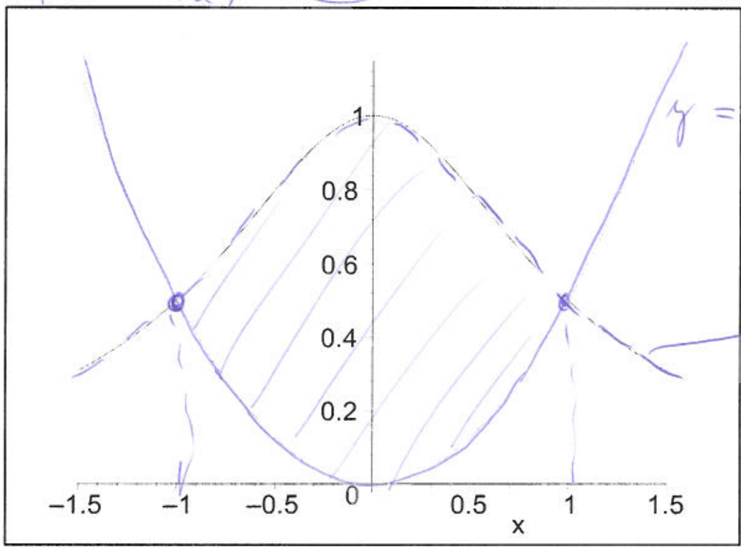
$\|$

$P_1 + P_2$

17

```
> solve(1/(1+x^2)=x^2/2,x);
> plot({1/(1+x^2),x^2/2},x=-1.5..1.5);
> int(1/(1+x^2)-x^2/2,x);
> int(1/(1+x^2)-x^2/2,x=-1..1);
```

Průsečíky $-1,\ 1,\ I\sqrt{2},\ -I\sqrt{2}$



$y = \frac{x^2}{2}$

$y = \frac{1}{1+x^2}$

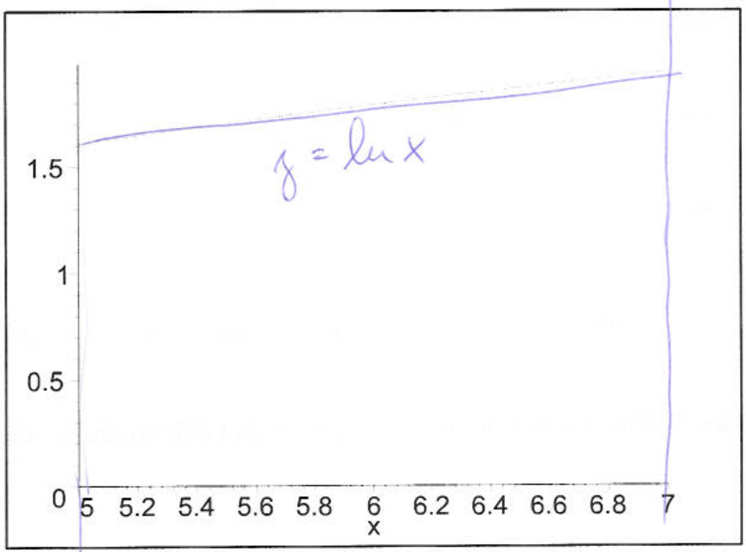
$\arctan(x) - \frac{1}{6}x^3$ PF na $\mathbb{R}$

$\frac{1}{2}\pi - \frac{1}{3} = \int_{-1}^{1}\left(\frac{1}{1+x^2} - \frac{x^2}{2}\right)dx$

8

x = 7

```
> plot({ln(x),0},x=5..7);
> int(ln(x),x);
> int(ln(x),x=5..7);
```



$$x\ln(x) - x$$

$$7\ln(7) - 2 - 5\ln(5)$$

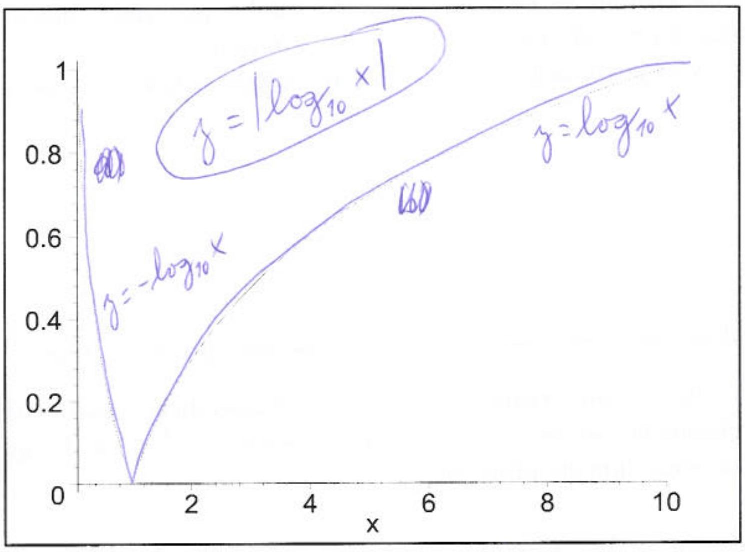
PF на $(0, +\infty)$

$$\int_5^7 \ln x \, dx$$

x = 5

9

```
> plot({abs(log[10](x)),0},x=1/10..10);
> int(abs(log[10](x)),x);
> int(abs(log[10](x)),x=1/10..10);
```



$$\frac{\begin{cases} a)\ -x\ln(x)+x & x\le 1 \\ b)\ x\ln(x)-x+2 & 1<x \end{cases}}{\ln(10)}$$

PF na $(0,+\infty)$

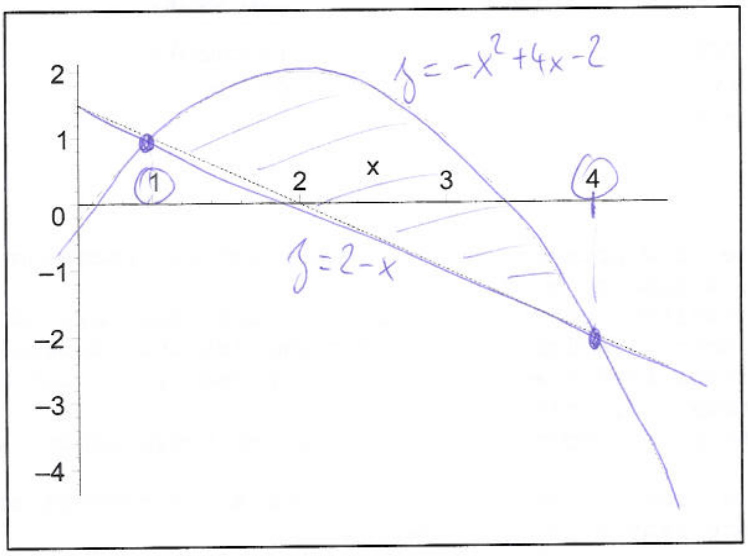
$$\frac{9}{10}\,\frac{11\ln(2)+11\ln(5)-9}{\ln(2)+\ln(5)} = \int_{1/10}^{10} |\log_{10} x|\,dx =$$

$$= \int_{1/10}^{1} (-\log_{10} x)\,dx + \int_{1}^{10} \log_{10} x\,dx =$$

$$= \left[\frac{-x\ln x + x}{\ln 10}\right]_{1/10}^{1} + \left[\frac{x\ln x - x + 2}{\ln 10}\right]_{1}^{10}$$

10

```
> plot({-x^2+4*x-2,2-x},x=0.5..4.5);
> solve(-x^2+4*x-2=2-x,x);
> int(-x^2+4*x-2-(2-x),x);
> int(-x^2+4*x-2-(2-x),x=1..4);
```



1, 4 Průsečíky

$-\frac{1}{3}x^3+\frac{5}{2}x^2-4x$ PF na $\mathbb{R}$

$\frac{9}{2}$

$$= \int_1^4 \left[(-x^2+4x-2)-(2-x)\right] dx =$$

$$= \int_1^4 (-x^2+5x-4)\,dx = \left[-\frac{x^3}{3}+\frac{5}{2}x^2-4x\right]_1^4 =$$

$$= \left(-\frac{64}{3}+40-16\right)-\left(-\frac{1}{3}+\frac{5}{2}-4\right) =$$

$$= -\frac{63}{3}+28-\frac{5}{2} = 7-\frac{5}{2} = \underline{\underline{\frac{9}{2}}} \; [j^2]$$