Písemka LS-2014 **P1–ℂ**

26. března 2014

KMA-MAT2/MT2

VZOR ....

15 1. Pro funkci $f: y = \dfrac{\ln(x^2+5x)}{x3^x}$ určete $D(f)$, f' a $D(f')$.

2. Vypočtěte následující integrály. Nezapomeňte na určení příslušných intervalů.

10 (a) $\int_{-2}^{2}(2|x| + |3x+12|)\,dx$, $= 56$

10 (b) $\int_0^1 x \ln x\,dx$, $= -\frac{1}{4}$

10 (c) $\int \cos^5 x \sin x\,dx$. $= -\frac{1}{6}\cos^6 x + C$

$P = 125/24 = 5\frac{5}{24}\ [j^2]$

15 3. Graficky znázorněte a vypočtěte obsah konečného rovinného obrazce ohraničeného křivkami: $y = 2x^2 - 2$ a $y = x + 1$.

① $D(f)$: $x^2+5x > 0 \wedge x \neq 0$

$x(x+5) > 0 \wedge x \neq 0$

$[(x>0 \wedge x>-5) \vee (x<0 \wedge x<-5)] \wedge x \neq 0$

$(x>0 \vee x<-5) \wedge x \neq 0$

$x \in (-\infty,-5) \cup (0,+\infty)$

$f'(x) = \dfrac{\left[\frac{1}{x^2+5x}\cdot(2x+5)\right]\cdot x\cdot 3^x - \ln(x^2+5x)\cdot\left[1\cdot 3^x + x\cdot 3^x \ln 3\right]}{(x3^x)^2}$

$D(f')$: $(x^2+5x \neq 0) \wedge (x^2+5x>0) \wedge x \neq 0$, tedy beze změny

$D(f') = D(f)$

2a $|x|$ n.b. $0 \in \langle -2,2\rangle$, $|3x+12|$ n.b. $-\frac{12}{3} = -4 \notin \langle -2,2\rangle$

$\int_{-2}^{2}(2|x|+|3x+12|)dx = 2\int_{-2}^{2}|x|dx + \int_{-2}^{2}|3x+12|dx = 2\int_{-2}^{0}(-x)dx + 2\int_0^2 x\,dx + \int_{-2}^{2}(3x+12)dx =$

$= 2\left[-\frac{x^2}{2}\right]_{-2}^{0} + 2\left[\frac{x^2}{2}\right]_0^2 + \left[\frac{3}{2}x^2+12x\right]_{-2}^{2} = [(-0)-(-4)] + [4-0] + [(6+24)-(6-24)] =$

$= 4+4+48 = 56$

2b $\int x \ln x\,dx = \begin{bmatrix} u=\ln x & v'=x \\ u'=\frac{1}{x} & v=\frac{x^2}{2}\end{bmatrix} = \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

$x>0$

$\int_0^1 x\ln x\,dx = \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_0^1 = \left(\frac{1^2}{2}\ln 1 - \frac{1^2}{4}\right) - \lim_{x\to 0}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right) =$

$= -\frac{1}{4} + 0 = -\frac{1}{4}$

$\lim_{x\to 0+}\frac{1}{2}x^2\cdot\ln x = [0\cdot(-\infty)] = \lim_{x\to 0+}\dfrac{\ln x}{2\frac{1}{x^2}} = \left[\frac{-\infty}{\infty}\right] \overset{LP}{=}$

$\overset{LP}{=} \lim_{x\to 0+}\dfrac{\frac{1}{x}}{2\cdot(-2)\frac{1}{x^3}} = \lim_{x\to 0+}\dfrac{x^2}{-4} = 0$

(2c) $\int \cos^5 x \sin x \, dx = \left[\begin{matrix} u = \cos x \\ du = -\sin x \, dx \\ -du = \sin x \, dx \end{matrix} \right] = \int u^5 (-du) = -\frac{u^6}{6} + C =$

$= -\frac{\cos^6 x}{6} + C$

(3) Průsečíky: $2x^2 - 2 = x + 1, \; 2x^2 - x - 3 = 0, \; D = 1 + 24 = 5^2$

$x_{1,2} = \frac{1 \pm 5}{4} = \left\langle \begin{matrix} \frac{3}{2} \\ -1 \end{matrix} \right.$

$0 \in \langle -1, \frac{3}{2} \rangle$

$2 \cdot 0^2 - 2 = -2$ $\quad 1 > -2 \Rightarrow$ $x+1$ má

$0 + 1 = 1$ na intervalu

$\langle -1, \frac{3}{2} \rangle$ větší hodnoty

než $2x^2 - 2$.

$$P = \int_{-1}^{\frac{3}{2}} \left[(x+1) - (2x^2 - 2) \right] dx =$$

$$= \int_{-1}^{\frac{3}{2}} \left[-2x^2 + x + 3 \right] dx = \left[-\frac{2}{3} x^3 + \frac{x^2}{2} + 3x \right]_{-1}^{\frac{3}{2}} =$$

$$= \left(-\frac{\cancel{2}}{\cancel{3}} \frac{\cancel{3} \cdot 9}{\cancel{2} \cdot 4} + \frac{\frac{9}{4}}{2} + 3 \cdot \frac{3}{2} \right) - \left(-\frac{2}{3} (-1)^3 + \frac{(-1)^2}{2} + 3(-1) \right) =$$

$$= -\frac{9}{4} + \frac{9}{8} + \frac{9}{2} - \frac{2}{3} - \frac{1}{2} + 3 = \frac{-\cancel{63}^{\,72} + 27 + 108 - 16 - 12 + 72}{24} =$$

$$= \frac{-\cancel{36}^{\,27} + 92 + 60}{24} = \frac{125}{24} = 5\frac{5}{24} \, [j^2]$$