Písemka LS-2014 **P1–D**

26. března 2014

KMA-MAT2/MT2

VZOR.............

15 1. Pro funkci $\quad f : y = \dfrac{\ln(x^2 - 5x)}{x2^x} \quad$ určete $D(f)$, $f'$ a $D(f')$.

2. Vypočtěte následující integrály. Nezapomeňte na určení příslušných intervalů.

10 (a) $\int_{-2}^{2} (2\,|x| + |3x - 12|)\,\mathrm{d}x, = 56$

10 (b) $\int_{0}^{1} x \ln x\,\mathrm{d}x, = -\frac{1}{4}$

10 (c) $\int \cos^5 x \sin^3 x\,\mathrm{d}x. = -\frac{1}{8}\sin^2 x \cos^6 x - \frac{1}{24}\cos^6 x + C$

$P = \frac{9}{2}$

15 3. Graficky znázorněte a vypočtěte obsah konečného rovinného obrazce ohraničeného křivkami:
$y = x^2 - 1$ a $\quad y = x + 1$.

$$f'(x) = \frac{\left[\frac{1}{x^2-5x}\cdot(2x-5)\right]\cdot x2^x - \ln(x^2-5x)\cdot\left[1\cdot 2^x + x\cdot 2^x \cdot \ln 2\right]}{(x2^x)^2}$$

$D(f'): (x^2-5x \neq 0) \wedge (x^2-5x > 0) \wedge x \neq 0$

$D(f') = D(f) = (-\infty, 0) \cup (5, +\infty)$

(1) $D(f): x^2 - 5x > 0 \wedge x \neq 0$

$x(x-5) > 0 \wedge x \neq 0$

$[(x>0 \wedge x-5>0) \vee (x<0 \wedge x-5<0)] \wedge x \neq 0$ spl.

$(x>0 \wedge x>5) \vee (x<0 \wedge x<5)$

$x>5 \quad \vee \quad x<0 \Rightarrow D(f) = (-\infty, 0) \cup (5, +\infty)$

(2a) $|x|$ u.b. $0 \in \langle -2, 2\rangle$, $\quad |3x-12|$ u.b. $\frac{12}{3} = 4 \notin \langle -2, 2 \rangle$

$-x \quad x \qquad -3x+12, \quad \cancel{3x-12}$

$$\int_{-2}^{2}(2|x|+|3x-12|)\,dx = \int_{-2}^{2} 2|x|\,dx + \int_{-2}^{2}|3x-12|\,dx = \int_{-2}^{0} 2(-x)\,dx + \int_{0}^{2} 2x\,dx + \int_{-2}^{2}(-3x+12)\,dx =$$

$$= \left[-x^2\right]_{-2}^{0} + \left[x^2\right]_0^2 + \left[-\frac{3}{2}x^2 + 12x\right]_{-2}^{2} = \qquad x \in \mathbb{R}.$$

$$= [(-0)-(-4)] + [4-0] + \left[\left(-\frac{3}{2}4 + 24\right) - \left(-\frac{3}{2}4 - 24\right)\right] = 4+4-6+24+6+24 = 56$$

(2b) $\int x \ln x\,dx = \begin{bmatrix} u = \ln x & v' = x \\ u' = \frac{1}{x} & v = \frac{x^2}{2} \end{bmatrix} = \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

$x \in (0, +\infty)$

$$\int_0^1 x \ln x\,dx = \frac{1}{2}\left[x^2\left(\ln x - \frac{1}{2}\right)\right]_0^1 = \frac{1}{2}\left[1^2\left(\ln 1 - \frac{1}{2}\right) - 0^2\left(\ln 0 - \frac{1}{2}\right)\right] = -\frac{1}{4}$$

nelze, musíme vypočítat limitu

$$\lim_{x\to 0^+} x^2 \cdot \ln x = [0 \cdot -\infty] = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \left[\frac{-\infty}{\infty}\right] \overset{LP}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x \to 0^+} \frac{1}{x}\cdot\frac{x^3}{-2} = \lim_{x\to 0^+}\frac{x^2}{-2} = 0$$

(2c) $\int \cos^5 x \sin^3 x \, dx = \int \cos^5 x \cdot \sin^2 x \cdot \sin x \, dx =$

$= \int \cos^5 x (1-\cos^2 x) \sin x \, dx = \left[ \begin{array}{l} u = \cos x \\ du = -\sin x \, dx \\ -du = \sin x \, dx \end{array} \right] = \int u^5 (1-u^2)(-du) =$

$= \int (u^5 - u^7)(-du) = \int (u^7 - u^5) du = \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$

$x \in \mathbb{R}$

(3) Průsečíky:

$x^2 - 1 = x + 1, \quad x^2 - x - 2 = 0 \quad D = 1 + 8 = 3^2$

$x_{1/2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases} \qquad 0 \in \langle -1, 2 \rangle$

$0^2 - 1 = -1 < 0 + 1 = 1 \Rightarrow$

$\Rightarrow x^2 - 1$ je na intervalu $\langle -1, 2 \rangle \leq$ než $x + 1$, tedy

$P = \int_{-1}^{2} [(x+1) - (x^2 - 1)] dx = \int_{-1}^{2} (-x^2 + x + 2) dx = \left[ -\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{2} =$

$= \left( -\frac{8}{3} + \frac{4}{2} + 4 \right) - \left( \frac{1}{3} + \frac{1}{2} - 2 \right) = -\frac{9}{3} + \frac{3}{2} + 6 = 4\frac{1}{2} = \frac{9}{2} \, [j^2]$