Využití L'Hospitalova pravidla pro výpočet limit funkcí

1. $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$ $\quad [1]$

2. $\lim\limits_{x\to 0} \dfrac{\mathrm{e}^x - 1}{x}$ $\quad [1]$

3. $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}$ $\quad [\ln a,\ a > 0, a \neq 1]$

4. $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x \sin x}$ $\quad \left[\dfrac{1}{2}\right]$

5. $\lim\limits_{x\to 1} \dfrac{\cos(\pi x) + 1}{(x-1)^2}$ $\quad \left[\dfrac{\pi^2}{2}\right]$

6. $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$ $\quad [0]$

7. $\lim\limits_{x\to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{\mathrm{e}^x - 1}\right)$ $\quad \left[\dfrac{1}{2}\right]$

8. $\lim\limits_{x\to 0^+} x \ln x$ $\quad [0]$

9. $\lim\limits_{x\to +\infty} \left(1 + \dfrac{1}{x}\right)^x = \left[\left(1 + \dfrac{1}{x}\right)^x = \mathrm{e}^{\ln\left(1+\frac{1}{x}\right)^x} = \mathrm{e}^{x \ln\left(1+\frac{1}{x}\right)}\right] = \mathrm{e}^{\lim\limits_{x\to+\infty} x \ln\left(1+\frac{1}{x}\right)}$ $\quad [\mathrm{e}]$

10. $\lim\limits_{x\to 0} (\cos x)^{\frac{1}{x^2}}$ $\quad \left[\dfrac{1}{\sqrt{\mathrm{e}}}\right]$

11. $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x}\right)^{\sin x}$ $\quad [1]$

12. $\lim\limits_{x\to 0} \dfrac{\mathrm{e}^x - 1}{\sin 2x}$ $\quad \left[\dfrac{1}{2}\right]$

13. $\lim\limits_{x\to 1} \dfrac{x - 1}{\ln x}$ $\quad [1]$

14. $\lim\limits_{x\to +\infty} \dfrac{\mathrm{e}^x}{x^3}$ $\quad [+\infty]$

15. $\lim\limits_{x\to 0^+} \dfrac{\ln x}{\operatorname{cotg} x}$ $\quad [0]$

16. $\lim\limits_{x\to 0} \dfrac{x - \sin x}{x^3}$ $\quad \left[\dfrac{1}{6}\right]$

17. $\lim\limits_{x\to +\infty} x\,\mathrm{e}^{-x}$ $\quad [0]$

18. $\lim\limits_{x\to +\infty} (\pi - 2 \operatorname{arctg} x) \ln x$ $\quad [0]$

19. $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \operatorname{cotg} x\right)$ $\quad [0]$

20. $\lim\limits_{x\to 0} (\cos 3x)^{\frac{1}{x^2}}$ $\quad \left[\mathrm{e}^{-\frac{9}{2}}\right]$

Neurčitý integrál — vzorečky

Funkce:	Funkce primitivní:	Funkce:	Funkce primitivní:
$x^m \quad (m \in \mathbb{R},\ m \neq -1)$	$\dfrac{x^{m+1}}{m+1}$	$\dfrac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x	a^x	$a^x/\ln a$
$\cos x$	$\sin x$	$\sin x$	$-\cos x$
$\dfrac{1}{\cos^2 x}$	$\operatorname{tg} x$	$-\dfrac{1}{\sin^2 x}$	$\operatorname{cotg} x$
$\dfrac{1}{1+x^2}$	$\operatorname{arctg} x$		

1. $\displaystyle\int 4x^{-3}\,\mathrm{d}x \qquad \left[-2x^{-2}+C\right]$

2. $\displaystyle\int \frac{x^3-2x+1}{x^3}\,\mathrm{d}x \qquad \left[x-\frac{1}{2x^2}+\frac{2}{x}+C\right]$

3. $\displaystyle\int \frac{50}{(5t)^3}\,\mathrm{d}t \qquad \left[-\frac{1}{5t^2}+C\right]$

4. $\displaystyle\int \left(\frac{1-x}{x}\right)^2\mathrm{d}x \qquad \left[x-2\ln\lvert x\rvert-\frac{1}{x}+C\right]$

5. $\displaystyle\int (x^3-3x^2+4x-7)\,\mathrm{d}x \qquad \left[\frac{x^4}{4}-x^3+2x^2-7x+C\right]$

6. $\displaystyle\int \left(1-\frac{1}{\sqrt[3]{x}}\right)^2\mathrm{d}x \qquad \left[x-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+C\right]$

7. $\displaystyle\int \sqrt{x}(1-x^2)\,\mathrm{d}x \qquad \left[\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{7}x^{\frac{7}{2}}+C\right]$

8. $\displaystyle\int \frac{2-x^2}{x-\sqrt{2}}\,\mathrm{d}x = \left[2-x^2=(\sqrt{2}-x)(\sqrt{2}+x)\right] \qquad \left[\sqrt{2}x-\frac{x^2}{2}+C\right]$

9. $\displaystyle\int (8\cos\alpha-3\sin\alpha)\,\mathrm{d}\alpha \qquad [8\sin\alpha+3\cos\alpha+C]$

10. $\displaystyle\int \left(\sin x-\frac{1}{\cos^2 x}\right)\mathrm{d}x \qquad [-\cos x-\operatorname{tg} x+C]$

11. $\displaystyle\int \frac{5\sin^2\Omega+3\cos^2\Omega}{2\sin^2\Omega\cos^2\Omega}\,\mathrm{d}\Omega \qquad \left[\frac{5}{2}\operatorname{tg}\Omega-\frac{3}{2}\operatorname{cotg}\Omega+C\right]$

12. $\displaystyle\int \frac{3-2\operatorname{cotg}^2 x}{\cos^2 x}\,\mathrm{d}x \qquad [3\operatorname{tg} x+2\operatorname{cotg} x+C]$

13. $\displaystyle\int \mathrm{e}^u\left(1+\frac{\mathrm{e}^{-u}}{\cos^2 u}\right)\mathrm{d}u \qquad [\mathrm{e}^u+\operatorname{tg} u+C]$

14. $\displaystyle\int \frac{5}{9+9t^2}\,\mathrm{d}t \qquad \left[\frac{5}{9}\operatorname{arctg} t+C\right]$

Neurčitý integrál — per partes

1. $\displaystyle\int x \sin x \,\mathrm{d}x \qquad [-x\cos x + \sin x + C]$

2. $\displaystyle\int x \cos x \,\mathrm{d}x \qquad [x\sin x + \cos x + C]$

3. $\displaystyle\int x^2 \sin x \,\mathrm{d}x \qquad \left[-x^2\cos x + 2x\sin x + 2\cos x + C\right]$

4. $\displaystyle\int \sin^2 x \,\mathrm{d}x \qquad \left[\frac{1}{2}(x - \sin x\cos x) + C\right]$

5. $\displaystyle\int x^2 \cos 2x \,\mathrm{d}x \qquad \left[\frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + C\right]$

6. $\displaystyle\int \mathrm{tg}^2\, x \,\mathrm{d}x = \int \sin^2 x \frac{1}{\cos^2 x}\,\mathrm{d}x \qquad \left[\frac{\sin^3 x}{\cos x} - x + \sin x\cos x + C\right]$

Neurčitý integrál — substituční metoda

1. $\displaystyle\int \frac{(1+\ln x)^4}{x}\,\mathrm{d}x = \left[\begin{array}{c} 1+\ln x = u \\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}u \end{array}\right] = \int u^4 \mathrm{d}u = \frac{1}{5}u^5 + C = \frac{1}{5}(1+\ln x)^5 + C$

2. $\displaystyle\int \sin x \cos x \,\mathrm{d}x \qquad \left[\frac{1}{2}\sin^2 x + C\right]$

3. $\displaystyle\int \sin^3 x \,\mathrm{d}x \qquad \left[-\cos x + \frac{1}{3}\cos^3 x + C\right]$

4. $\displaystyle\int \frac{\ln^5 x}{x}\,\mathrm{d}x \qquad \left[\frac{1}{6}\ln^6 + C\right]$

5. $\displaystyle\int \frac{\mathrm{e}^{\mathrm{tg}\, x}}{\cos^2 x}\,\mathrm{d}x \qquad \left[\mathrm{e}^{\mathrm{tg}\, x} + C\right]$

6. $\displaystyle\int \frac{31\sin(\mathrm{arctg}\, x)}{1+x^2}\,\mathrm{d}x \qquad [-31\cos(\mathrm{arctg}\, x) + C]$