Definition of the Choquet integral with respect to fuzzified fuzzy measure

Ondřej Pavlačka
Department of Mathematical Analysis and Applications of Mathematics,
Faculty of Science, Palacký University Olomouc,
tř. 17. listopadu 12, 771 46 Olomouc, Czech Republic,
ondrej.pavlacka@upol.cz

Definition 1 Let $\Omega = \{\omega_1, \ldots, \omega_n\}$ be a nonempty finite set, $B_1, \ldots, B_{2^n-1}$ be all its nonempty subsets, μ_F be a fuzzified fuzzy measure on Ω, and $F : \Omega \to \mathcal{F}_N([0,1])$, $F(\omega_i) = H_i$, $i = 1, \ldots, n$ be a FNV-function. *The Choquet integral of F with respect to fuzzified fuzzy measure μ_F* is defined as a fuzzy number H with a membership function given for any $h \in [0,1]$ by

$$\begin{aligned} H(h) \;=\; & \max\Big\{ \min\{H_1(h_1), \ldots, H_n(h_n), \mu_F(B_{(1)})(\mu_1), \ldots, \mu_F(B_{(n)})(\mu_n)\} \mid \\ & h = (C)\int_\Omega f d\mu, \text{ where } f : \Omega \to [0,1] \text{ such that } f(\omega_i) = h_i, \\ & i = 1, \ldots, n, \text{ and } \mu \text{ is a fuzzy measure on } \Omega \text{ such that } \mu(B_{(i)}) = \mu_i, \\ & i = 1, \ldots, n, \text{ where } (i) \text{ is a permutation such that } h_{(1)} \leq \ldots \leq h_{(n)}, \\ & \text{and } B_{(i)} = \{\omega_{(i)}, \ldots, \omega_{(n)}\}\Big\}. \end{aligned}$$